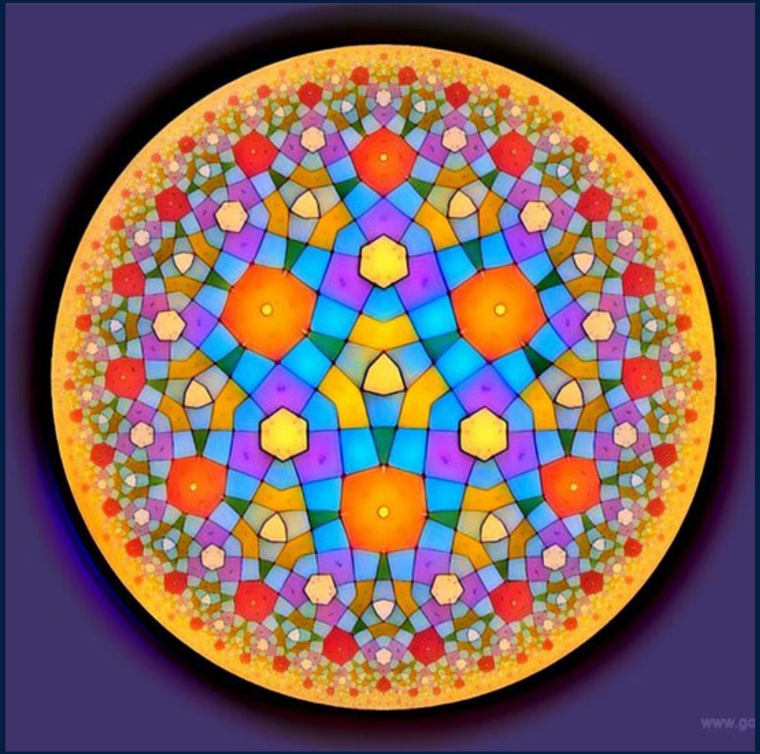
THE UNIVERSITY of EDINBURGH

Hyperbolic Explorations



Cordelia Webb

Open to the world

10.43. *Given a ray* p *and a line* **r** *which is neither parallel nor perpendicular to* p, *construct the line which is parallel to* p *and perpendicular to* **r**.

CONSTRUCTION.* Draw p′, the image of p by reflection in **r**. Then, clearly, the desired line is the common parallel to p and p′.

10.5. An alternative expression for distance. It is sometimes convenient to replace 10.31 by a formula resembling 5.76 instead of 6.91. Such a formula can most easily be obtained by using *abscissae*, with the involution of conjugate points in the form

$$x + x' = 0$$

(which is 4.29 with $s = 0$), so that the points at infinity, **M** and **N**, have abscissae 0 and ∞, while ordinary points have positive abscissae. Let x and y be the abscissae of **A** and **B**, so that $-x$ and $-y$ are those of their respective conjugates, **A**′ and **B**′. Then, in terms of Lobatschewsky's unit of measurement, we have, by 4.33,

$$e^{\pm 2\mathbf{AB}} = \{\mathbf{AB}, \mathbf{MN}\} = \frac{x}{y},$$

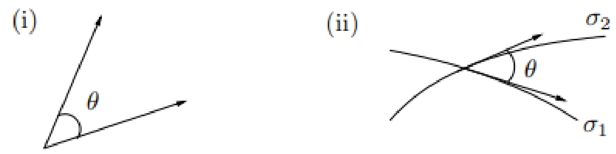
whence

$$\begin{aligned} \cosh \mathbf{AB} &= \tfrac{1}{2}(e^{\mathbf{AB}} + e^{-\mathbf{AB}}) \\ &= \frac{1}{2}\left(\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}}\right) \\ &= \frac{x+y}{2\sqrt{(xy)}} \\ &= \sqrt{\frac{(x+y)\,(y+x)}{(x+x)\,(y+y)}} \\ &= \sqrt{\{\mathbf{AB}, \mathbf{B'A'}\}}\,. \end{aligned}$$

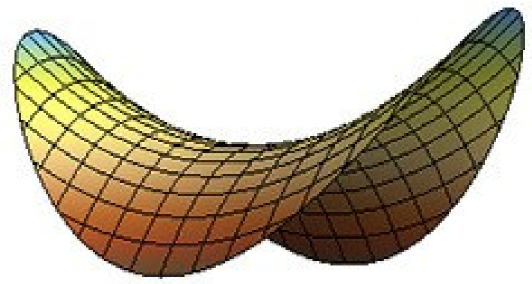
*Carslaw [1], p. 76.

§5.6 Angles

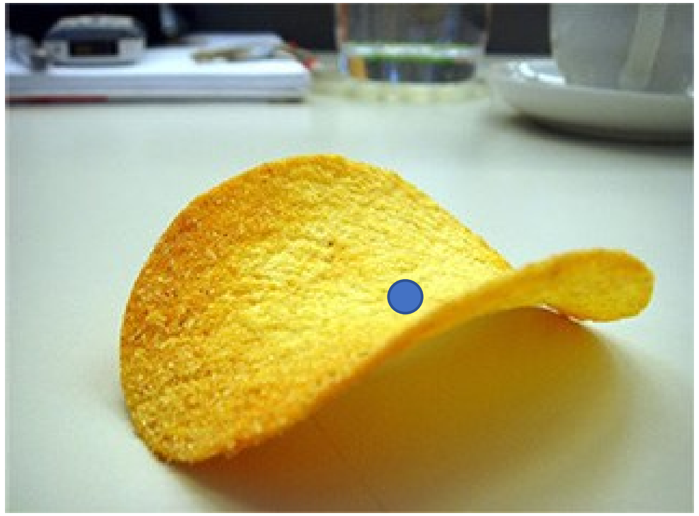
Suppose that we have two paths σ_1 and σ_2 that intersect at the point $z \in \mathbb{H}$. By choosing appropriate parametrisations of the paths, we can assume that $z = \sigma_1(0) = \sigma_2(0)$. The angle between σ_1 and σ_2 at z is defined to be the angle between their tangent vectors at the point of intersection and is denoted by $\angle\sigma_1'(0), \sigma_2'(0)$,

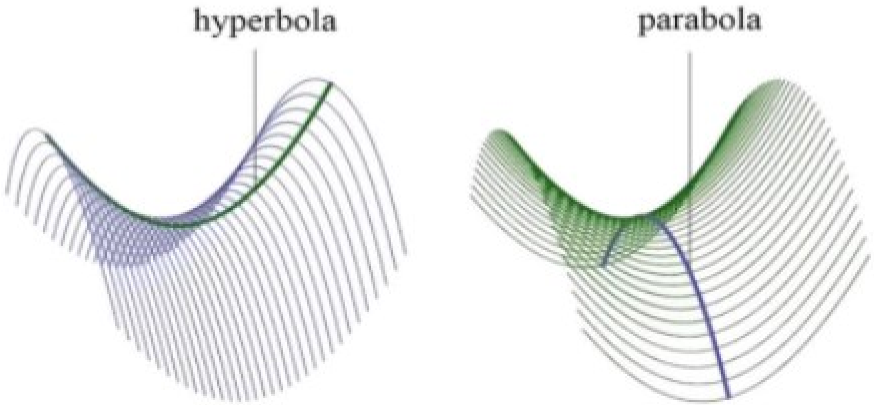


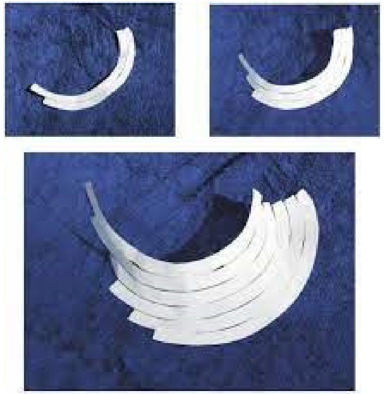
Hyperbolic Paraboloid



$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$$

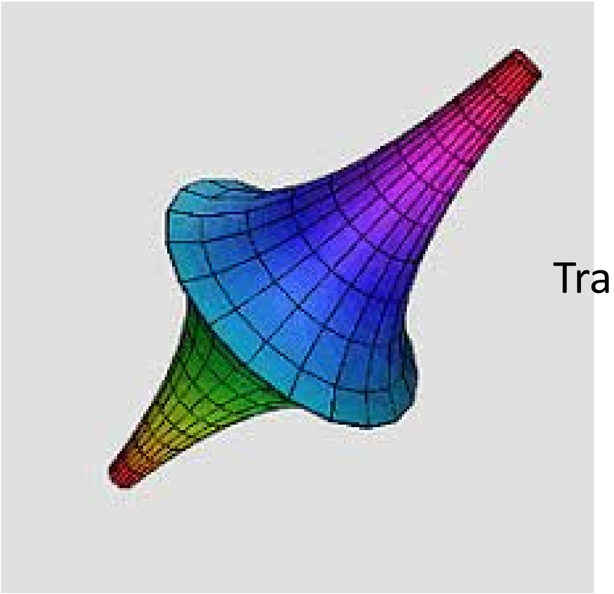




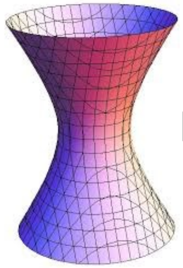


Pseudosphere

Tractrix



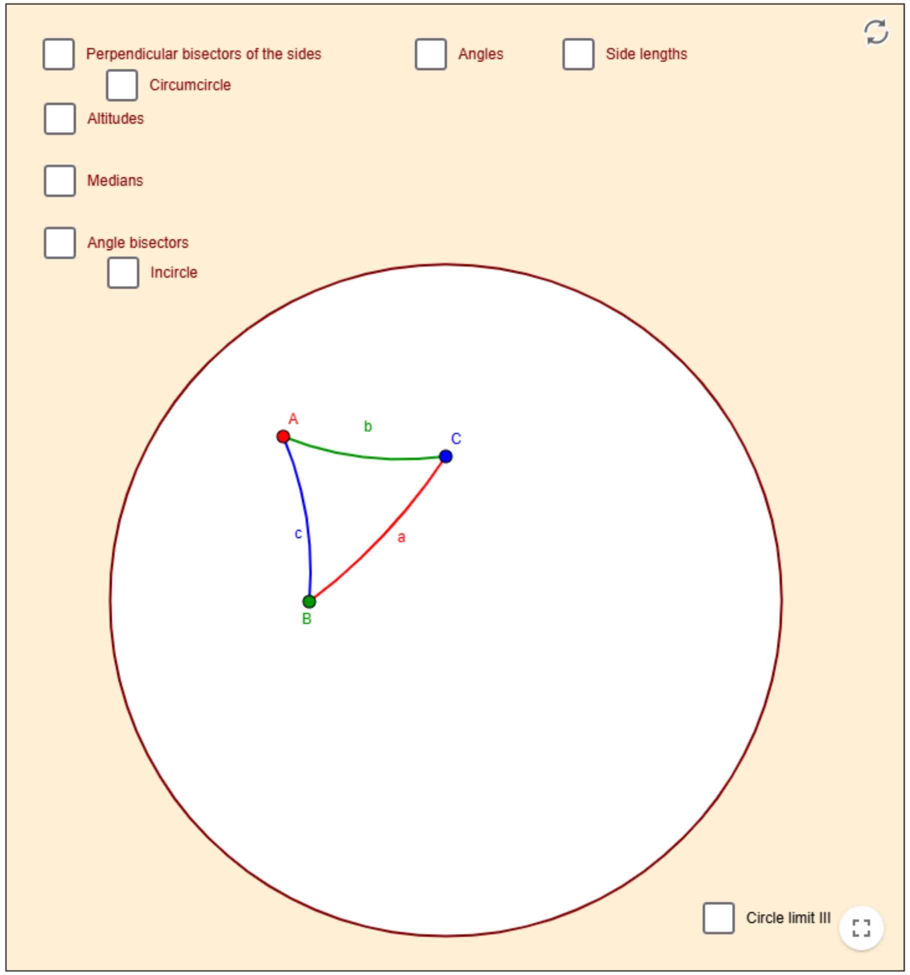
Tractroid



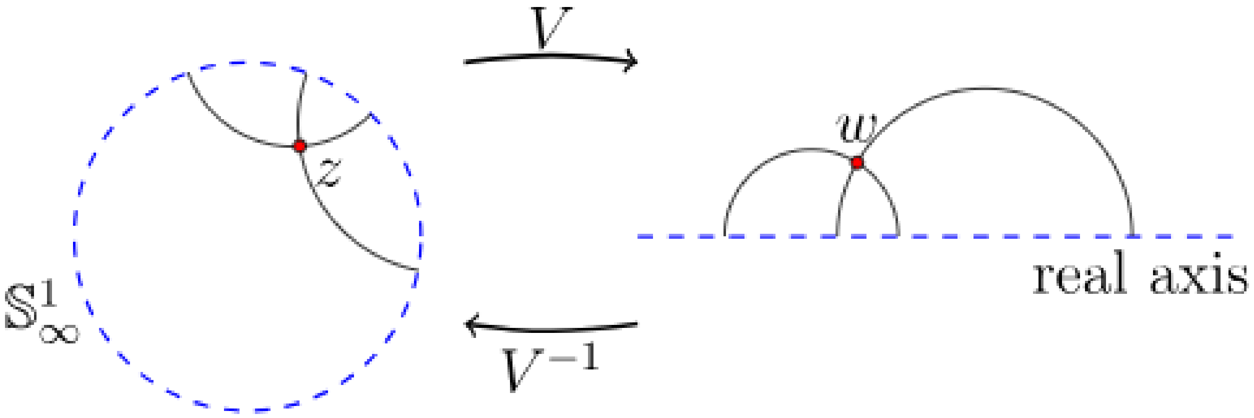
Hyperboloid

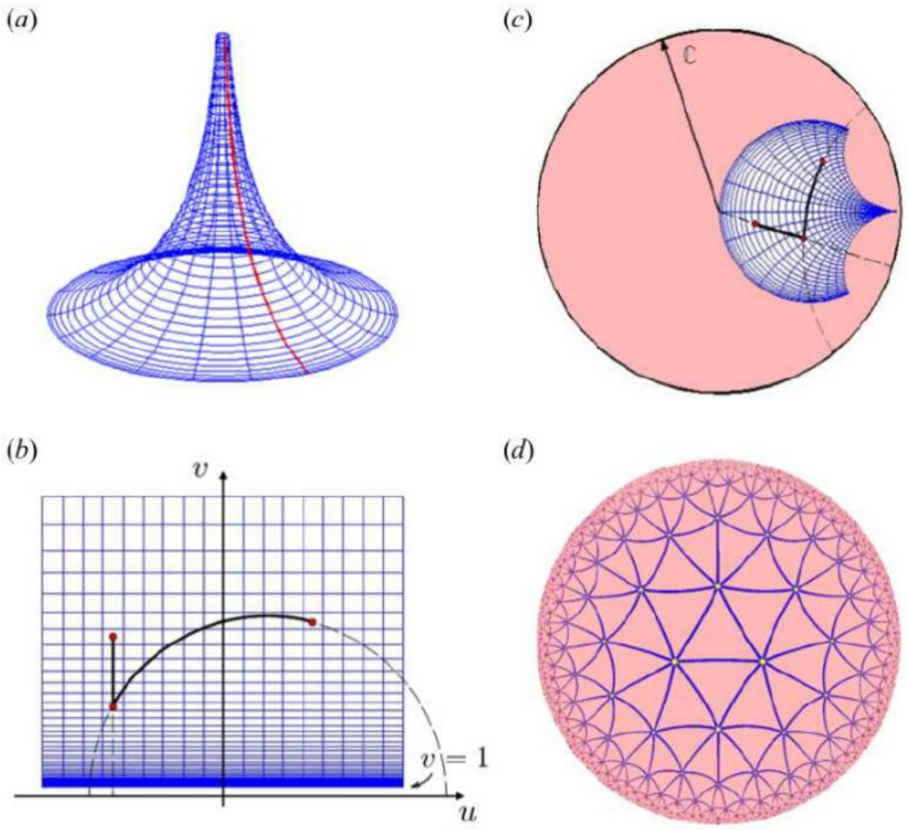
Lines and angles

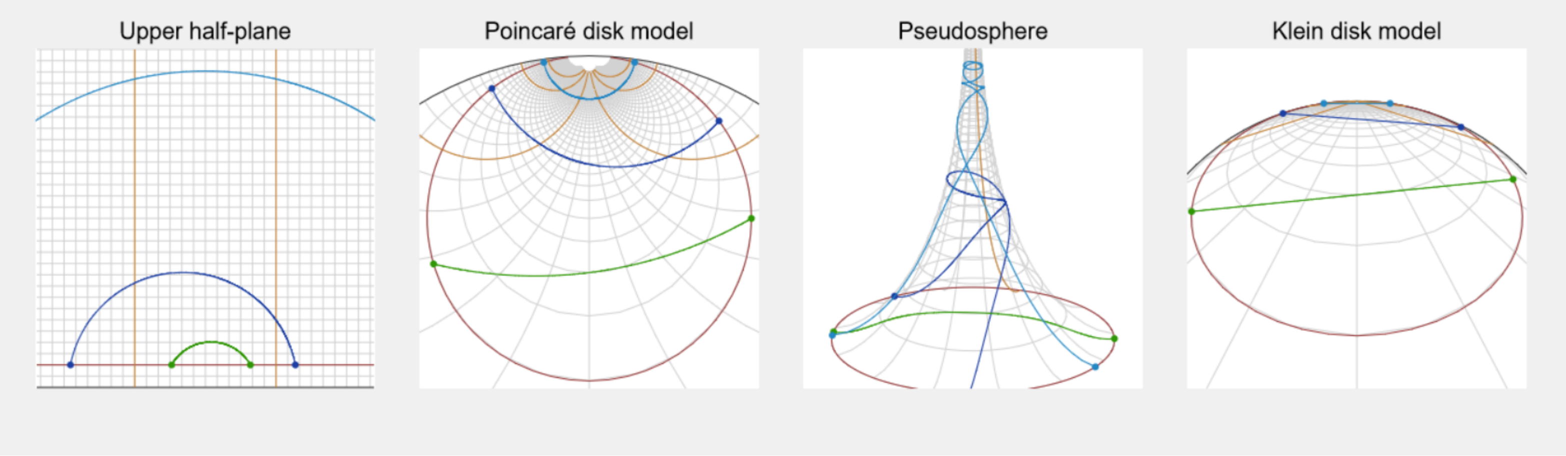
Poincare disc model



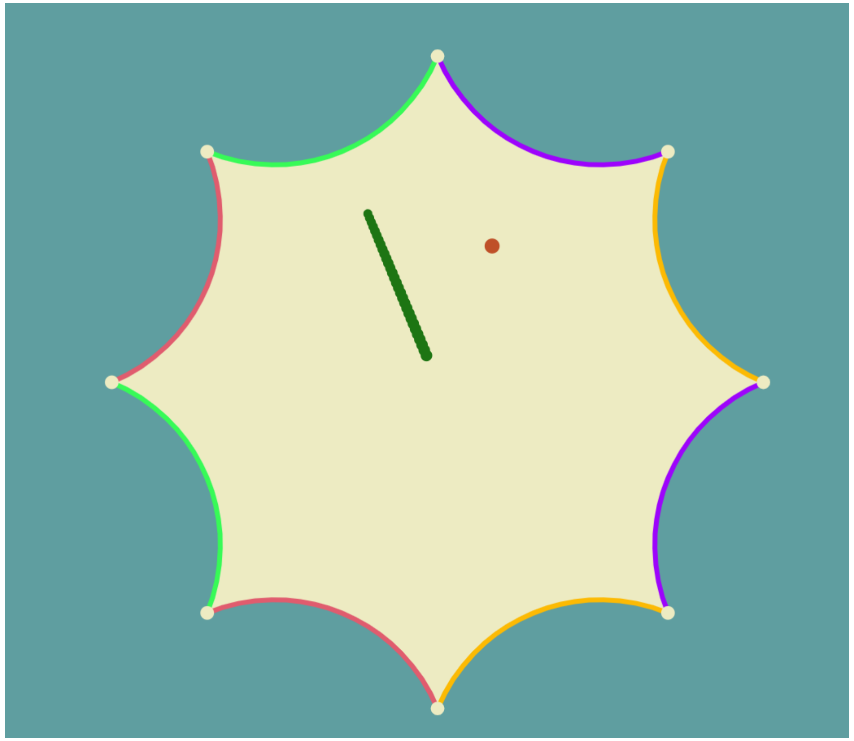
Other models

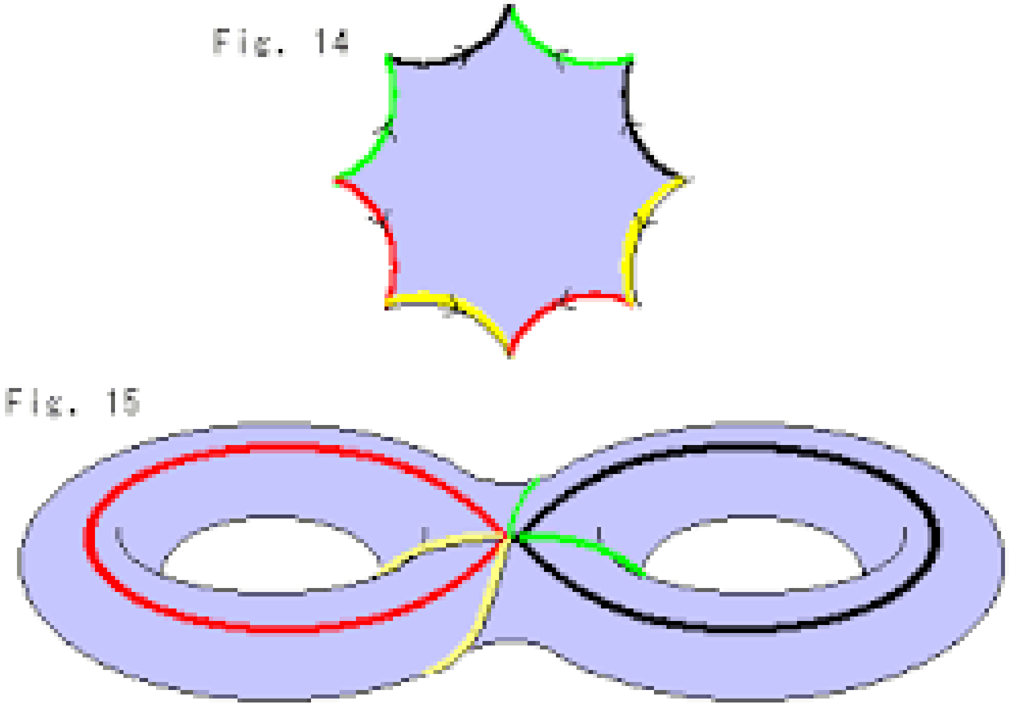




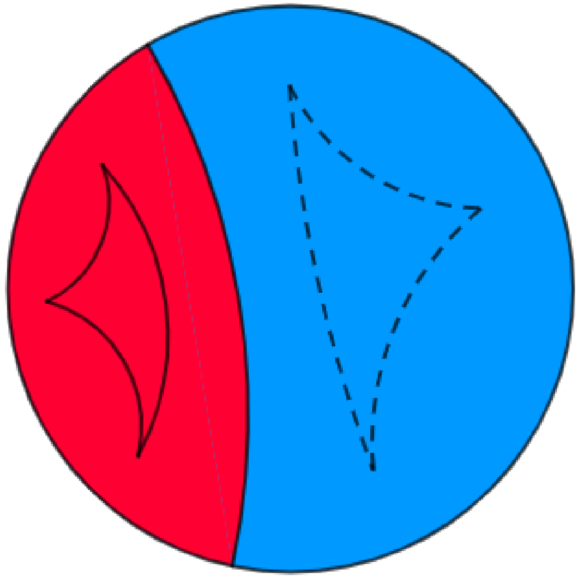


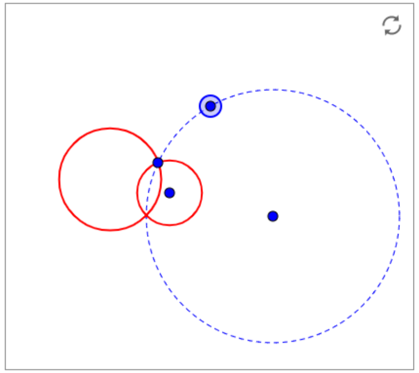
Surfaces to Uniformisation

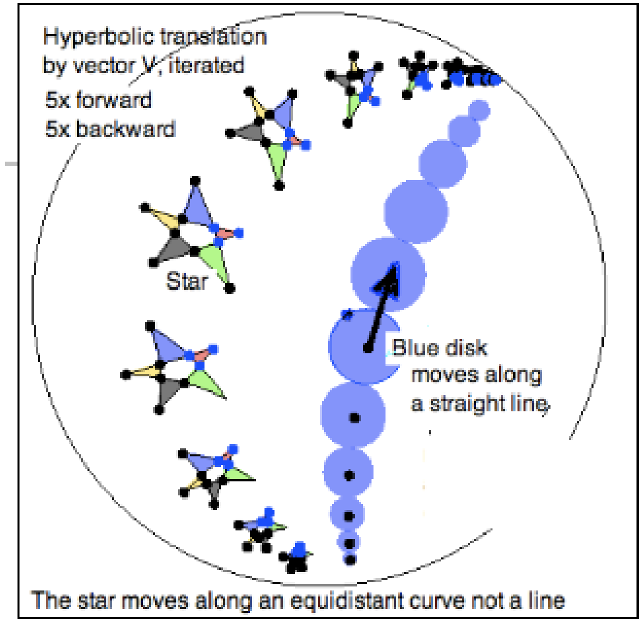


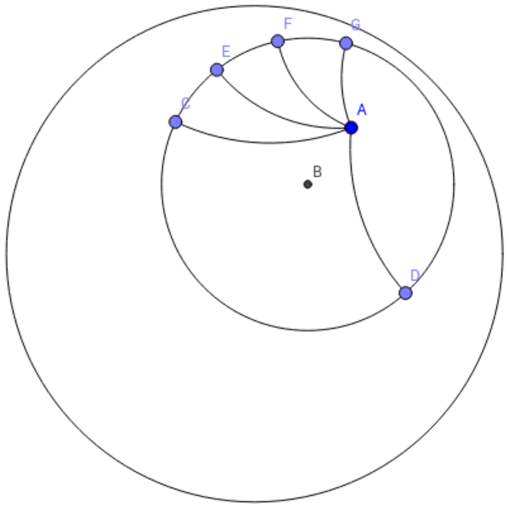


Isometries

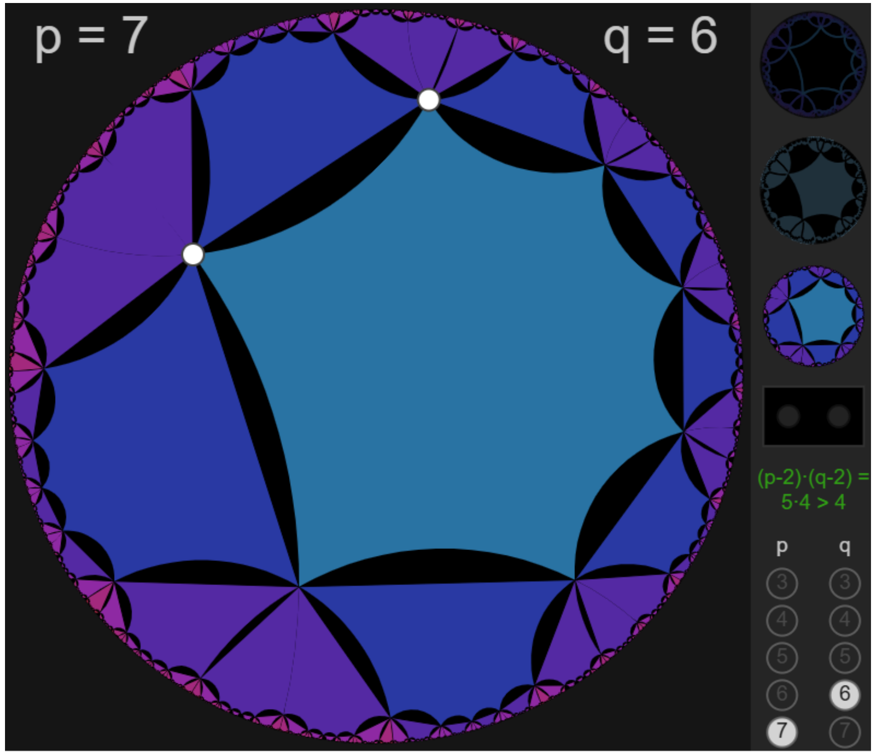


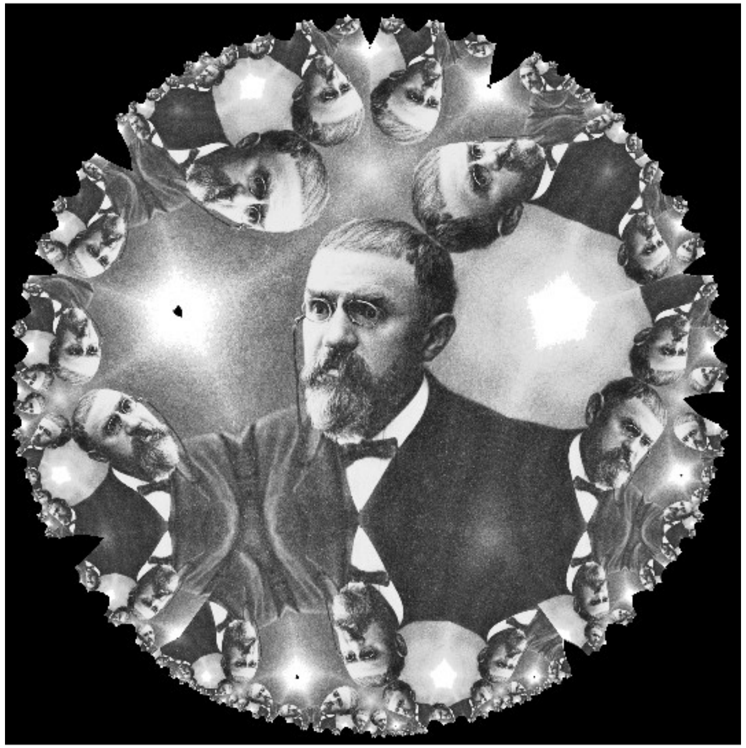


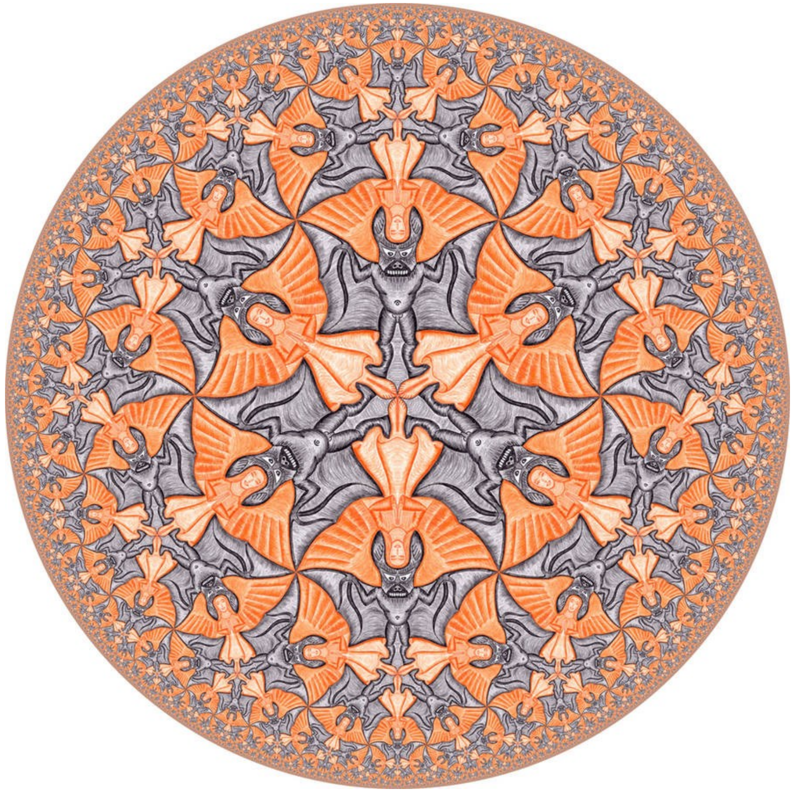

Hyperbolic translation
by vector V, iterated
5x forward
5x backward
Star
Blue disk
moves along
a straight line
The star moves along an equidistant curve not a line


C
E
F
G
A
B
D

Tilings







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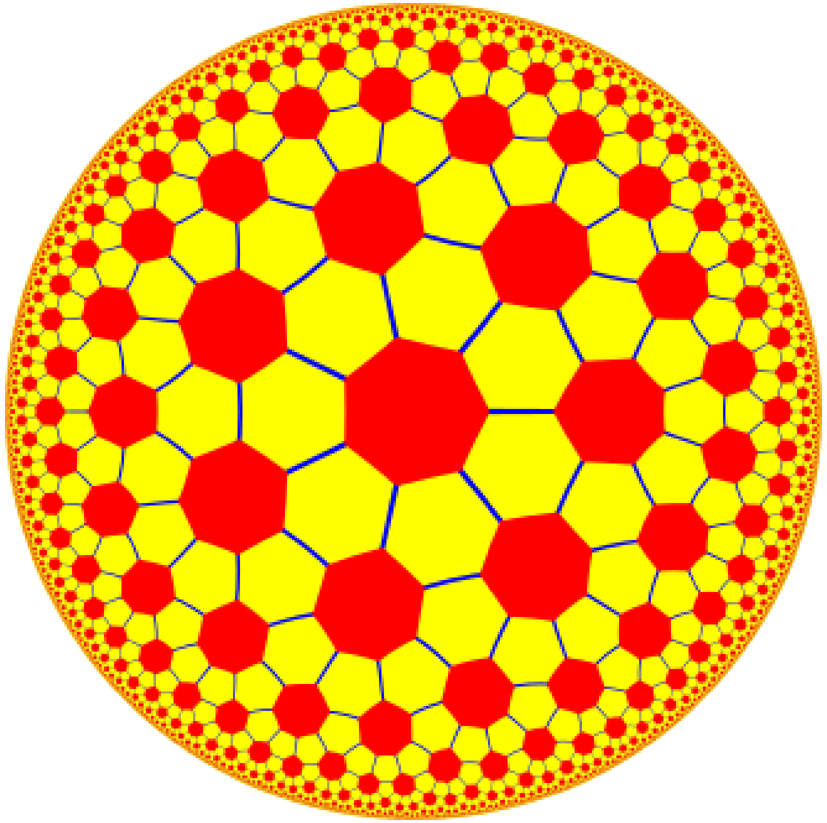

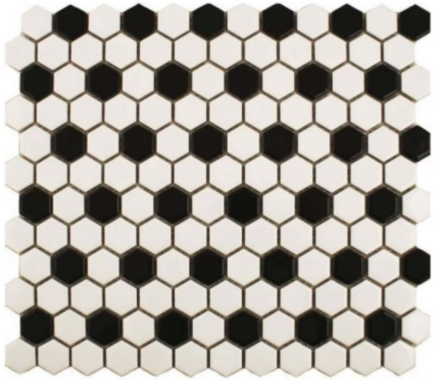

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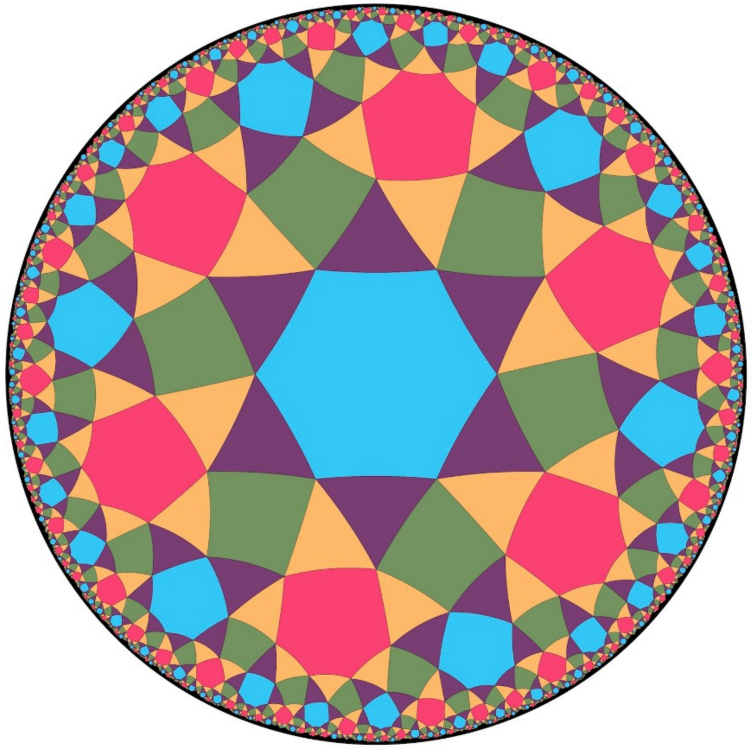
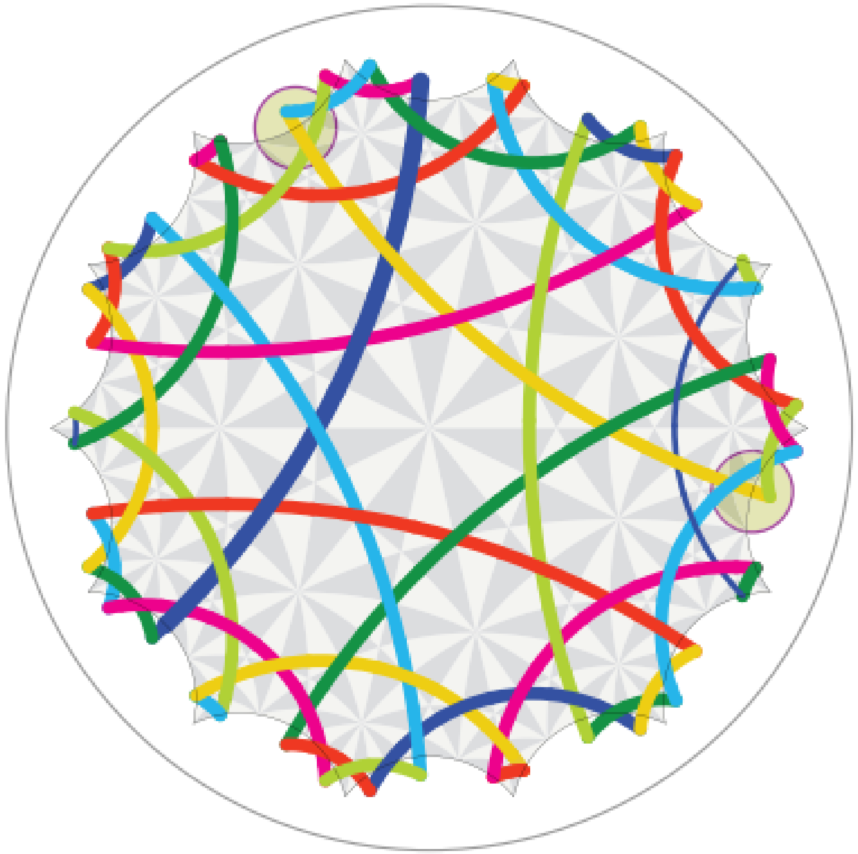
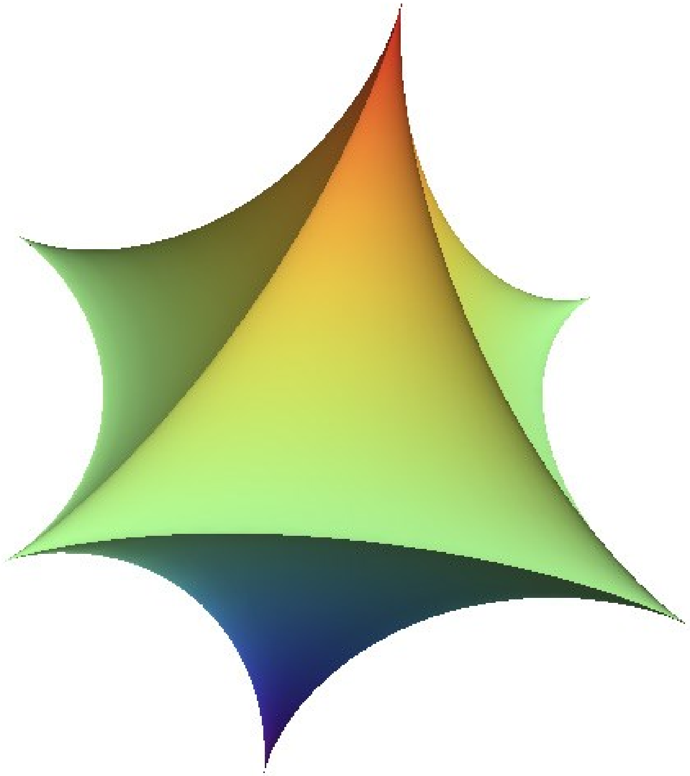
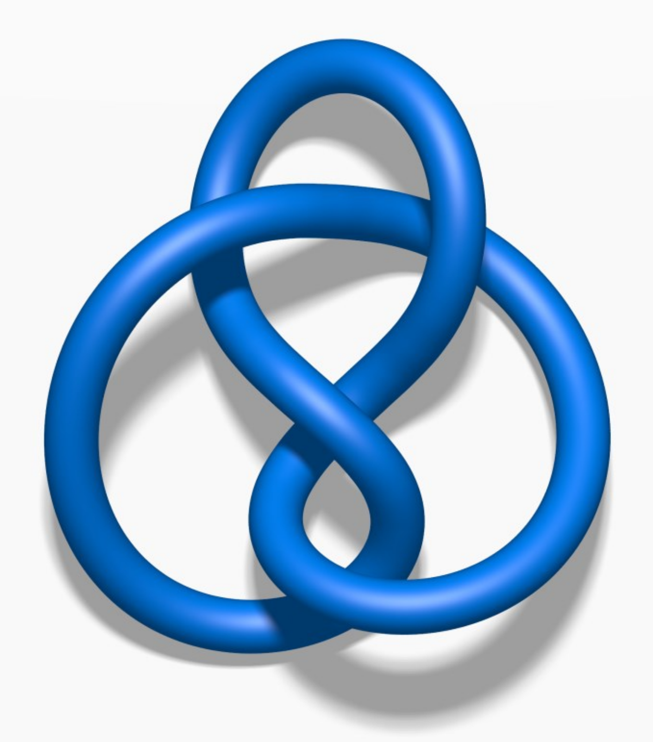


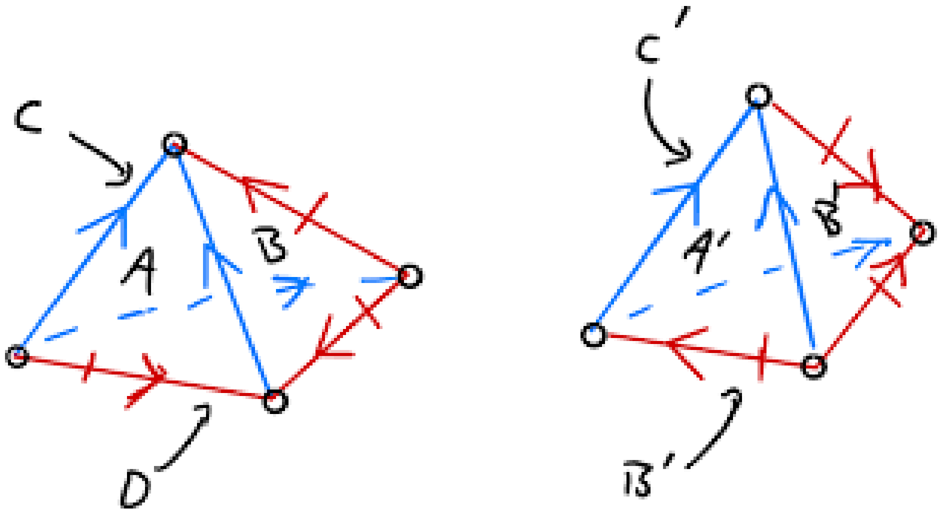
Hyperbolic Polyhedron

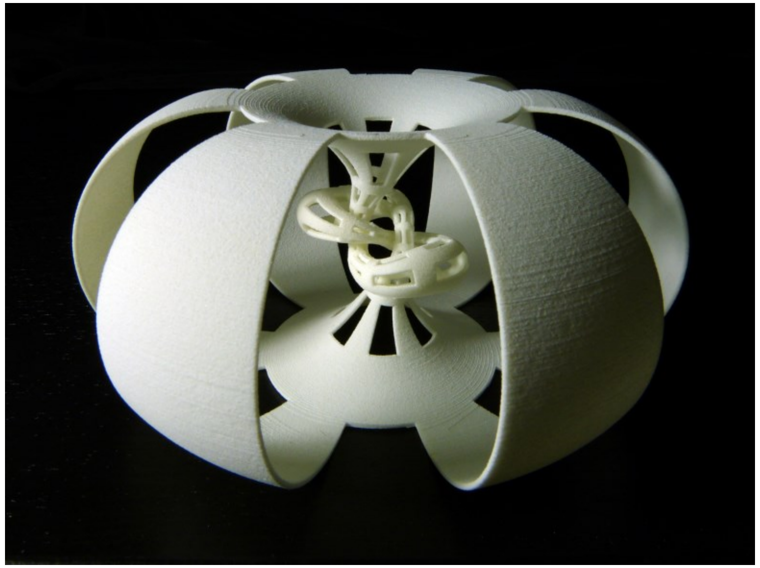




Knot Theory







Thank you!