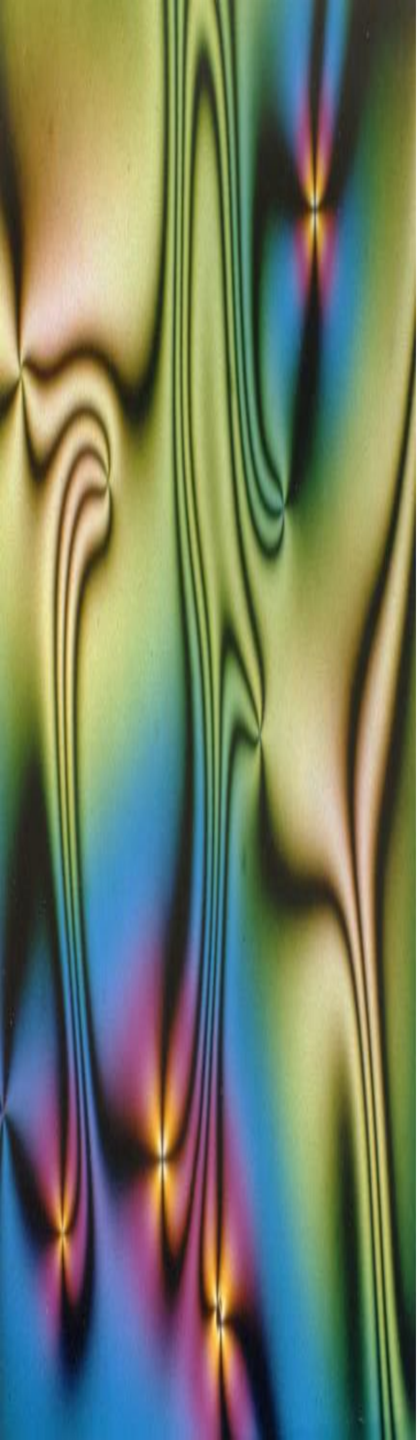


THE MATHEMATICS OF LIQUID CRYSTALS: ANALYSIS, MODELLING AND COMPUTATIONS

APALA MAJUMDAR
UNIVERSITY OF STRATHCLYDE

PiWORKS SEMINAR
UNIVERSITY OF STRATHCLYDE
25 MARCH 2025

Brief Outline of Talk

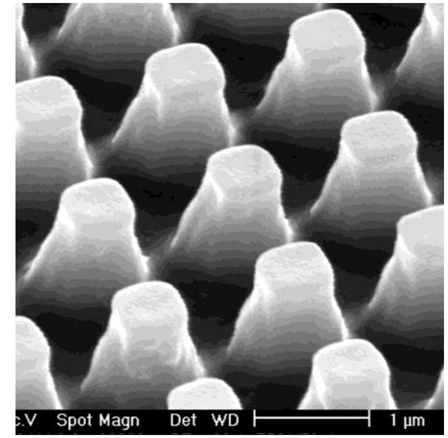
- ✓ Career Milestones
- ✓ Legacy of Strathclyde in Liquid Crystals
- ✓ Overview of Liquid Crystals and Research Programme
- ✓ Continuum Mechanics and Industrial Mathematics (CMIM) Research Group
- ✓ Some Reflections on Careers in Mathematics

University of Bristol (Undergraduate and PhD)

- MSci in Mathematics and Physics (4 years)

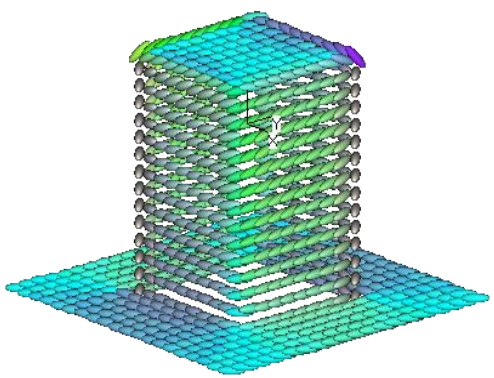
- PhD in Applied Mathematics

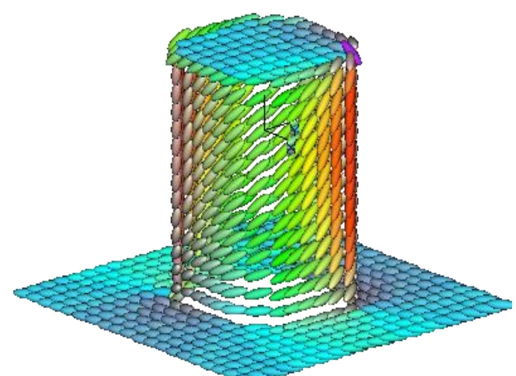
Title of Thesis " *Liquid Crystals and tangent unit-vector fields in polyhedral geometries*"



- CASE (Competitive Award in Science and Engineering) with Hewlett Packard – worked on a real bistable device called the Post Aligned Bistable Nematic Device.

Kitson and Geisow, Applied Physics Letters, 80,2002.





Numerical modelling by Chris Newton (HP Labs)

University of Oxford (Research Fellowships)

- ❑ Royal Commission for the Exhibition of 1851 Fellowship (2 years)

Oxford Centre for Nonlinear Partial Differential Equations

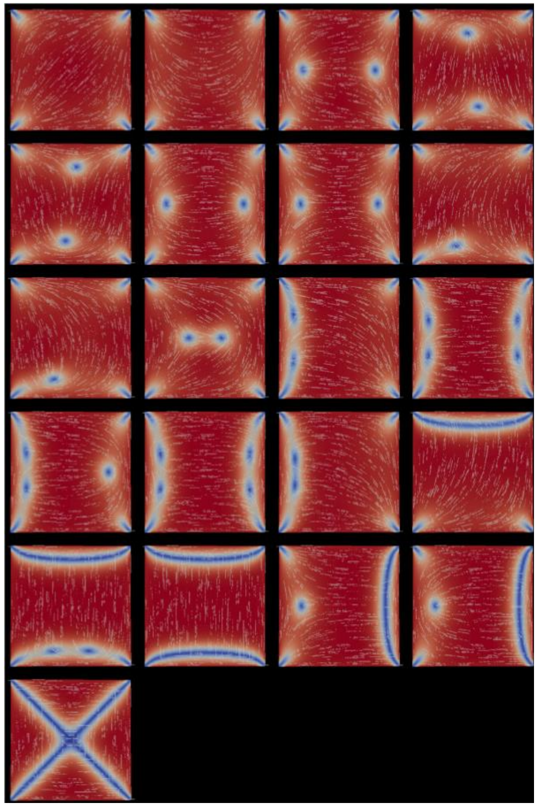
- ❑ Oxford Centre for Collaborative Applied Mathematics (4 years)

- ❑ First PhD student (Alexander Lewis)

- ❑ Interdisciplinary Research

- ❑ International Collaborations

- ❑ International Workshop Organization (2009 and 2012)



University of Bath (Faculty Position)

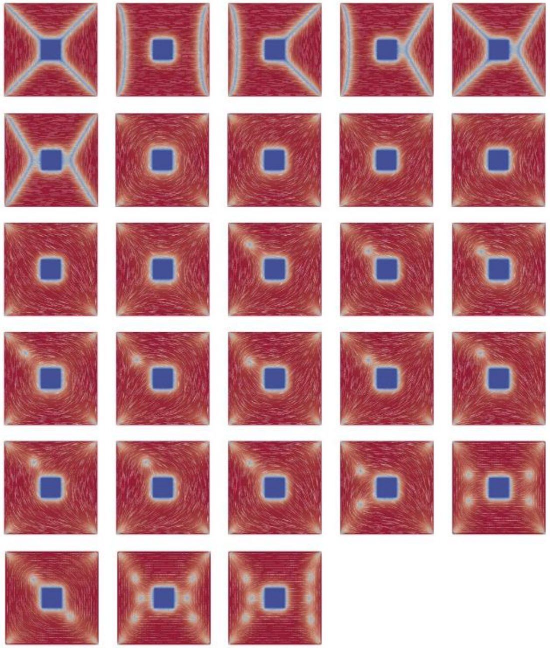
- ❑ Research, Teaching, Management & Leadership

- ❑ Director for Centre for Nonlinear Mechanics (1 year)

- ❑ Principal Investigator for Bath-Chile-Mexico network (University of Bath, Two Universities in Chile (CMM and Catolica University), UNAM and CIMAT in Mexico)

➢ Co-organised two workshops in Chile!

➢ Huge exposure to Internationalisation

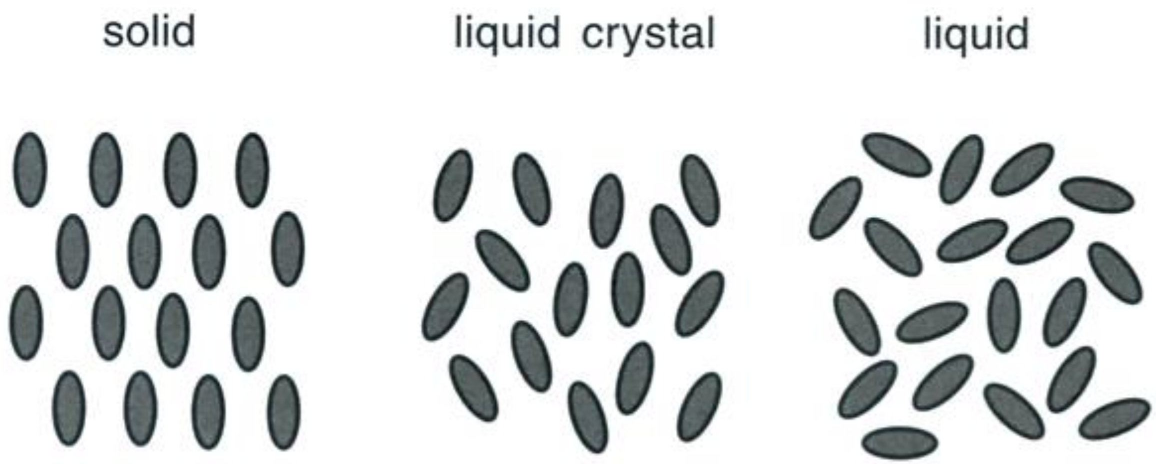


Y. Wang, G. Canevari and A. Majumdar, *Order Reconstruction for Nematics on Squares with Isotropic Inclusions : A Landau-de Gennes Study*, SIAM Journal of Applied Mathematics, 79 (2019), 1314.

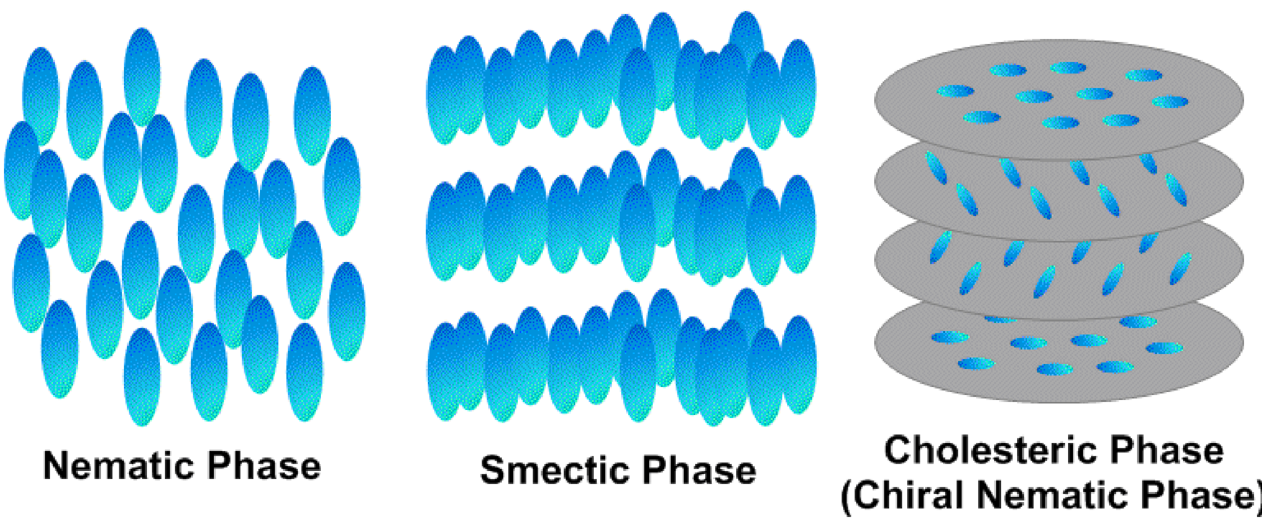
University of Strathclyde

- Global Talent Attraction Platform (GTAP) Professor of Applied Mathematics
- Associate Dean for International Research, Faculty of Science (2022-2024)
- Regional Lead Coordinator for South Asia Region
- EDI Convenor for Mathematics and Statistics
- Retreat for Women in Applied Mathematics Event (ICMS, Edinburgh)

Liquid Crystals – what are they?



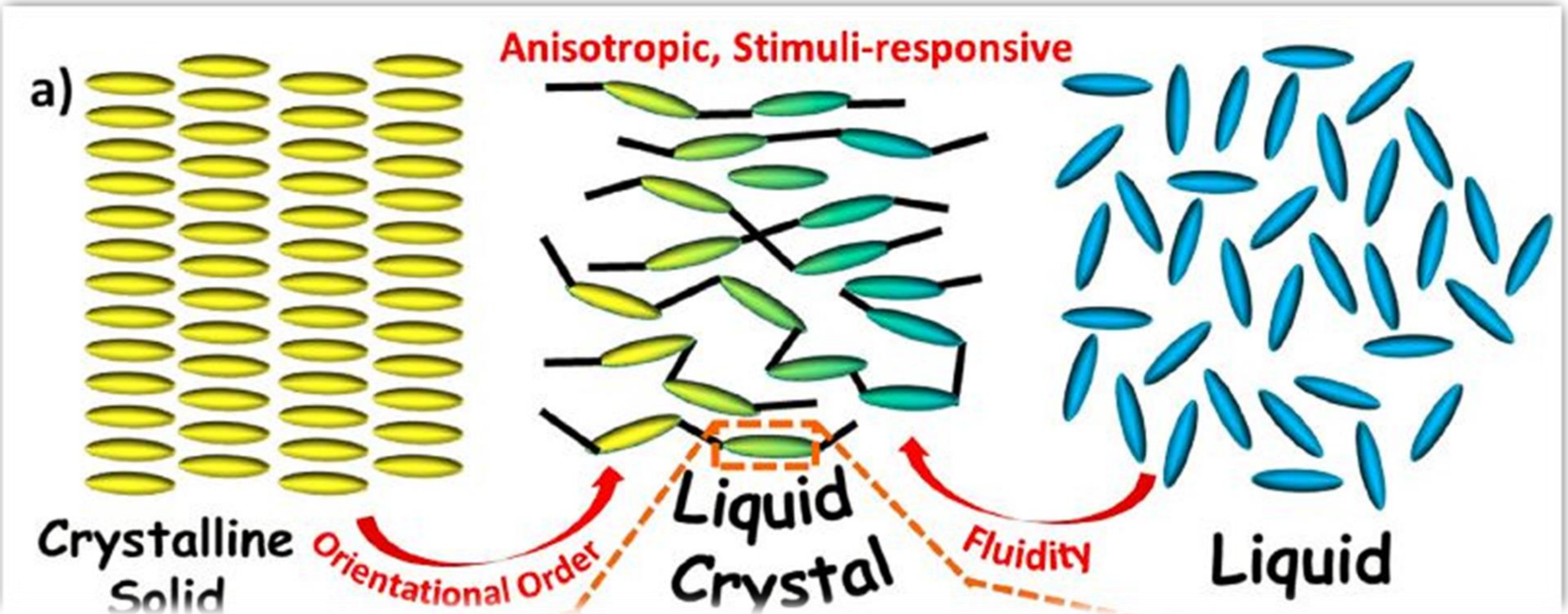
(Felix, et al., 2015)



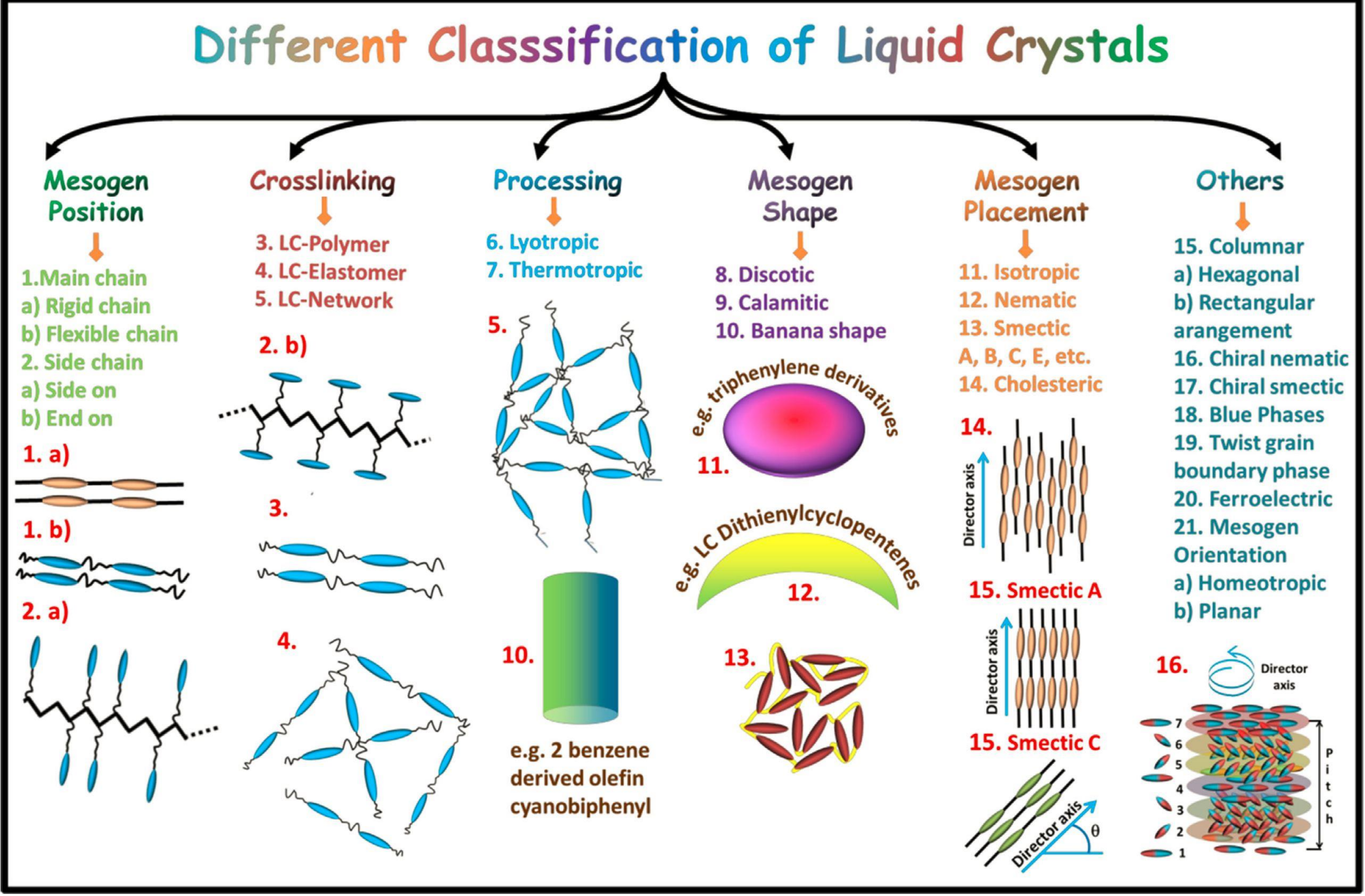
(tokyo chemical industry)

What do I work on?

- **Mathematics of Liquid Crystals and Their Applications**

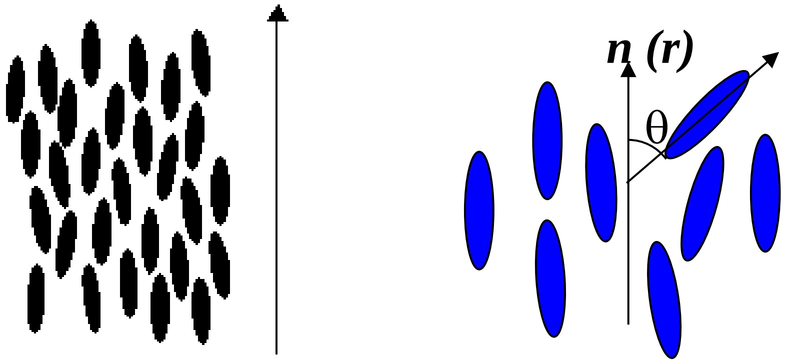


R. Prasansha, et. al. European Polymer Journal 121 (2019): 109287.

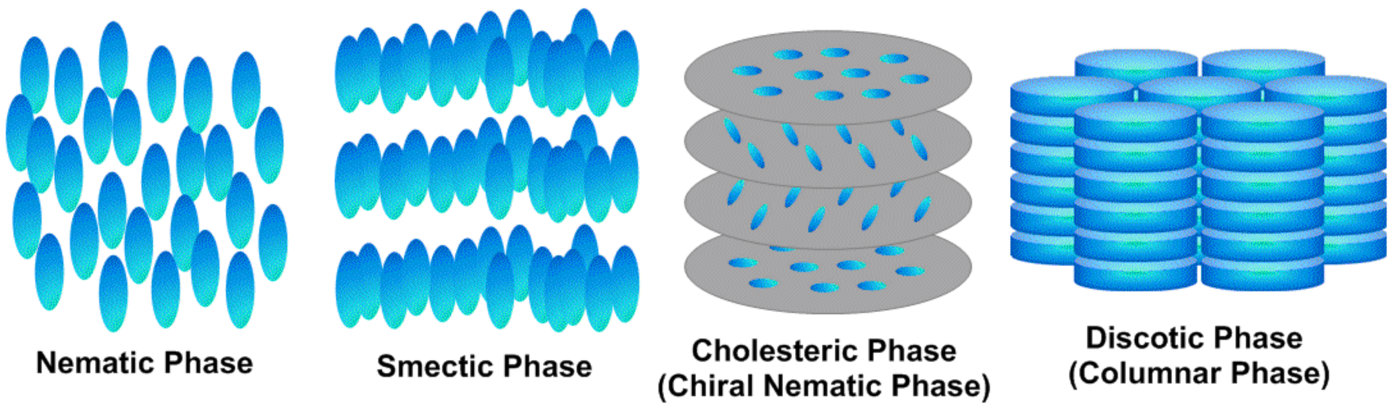


R. Prasansha, et. al. European Polymer Journal 121 (2019): 109287.

“Orientationally ordered soft matter is exceptionally responsive to a variety of excitations. That’s the basis for its great range of applications.”



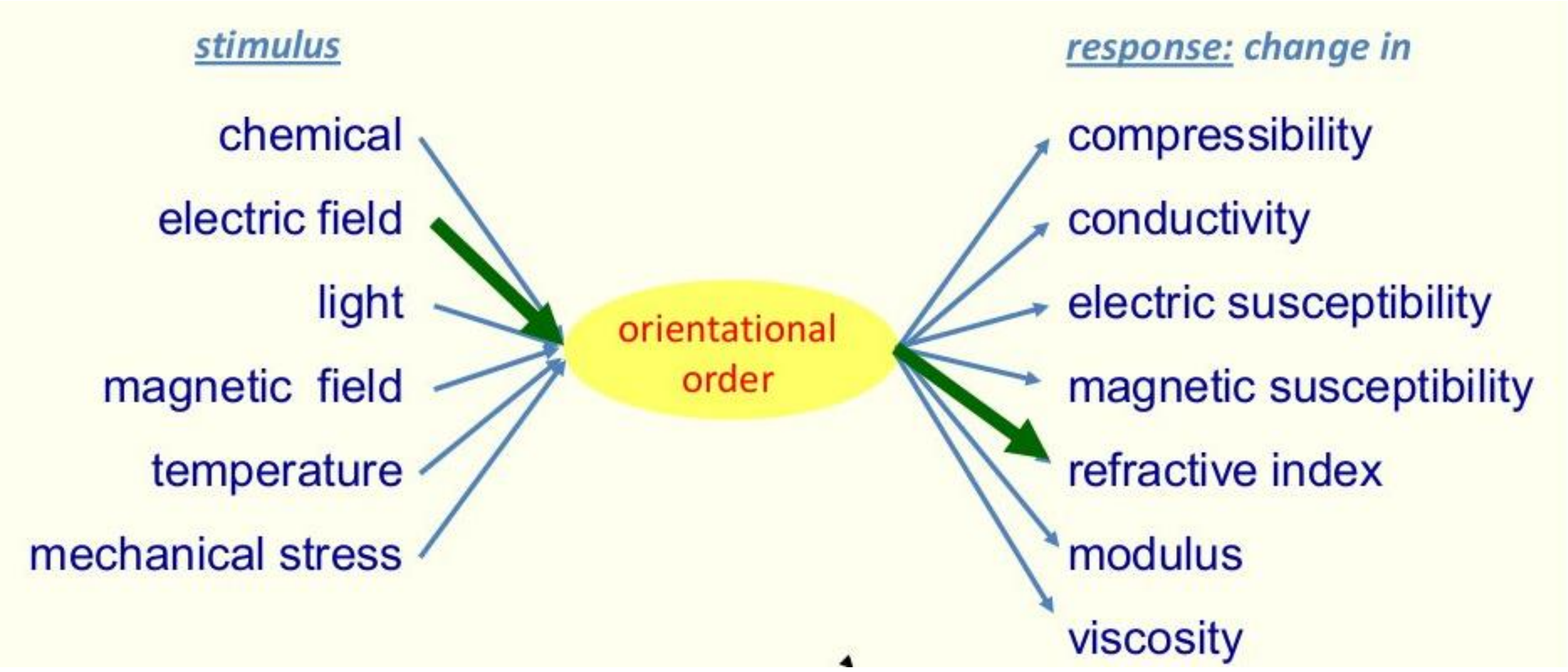
n (r) : preferred direction of orientation of the long molecular axes.



(tokyo chemical industry)

Peter Palffy-Muhoray, The diverse world of Liquid Crystals. Physics Today, 2007.
Jan Lagerwall, https://www.lcsoftmatter.com/

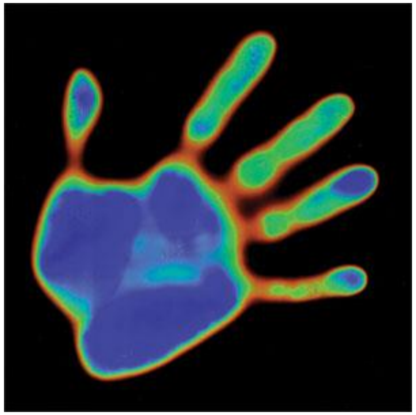
Key word: anisotropy!!!

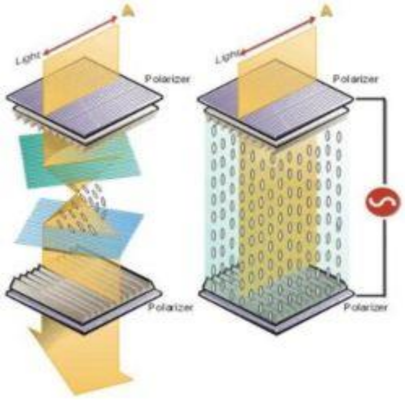


Courtesy: Images from Peter Palffy-Muhoray's lectures at Colorado – Boulder (*Physics Today 60 (9), 54 (2007))*

Applications

- Soft materials: weak perturbations lead to mesoscopic or macroscopic responses $\Rightarrow$ array of applications !!



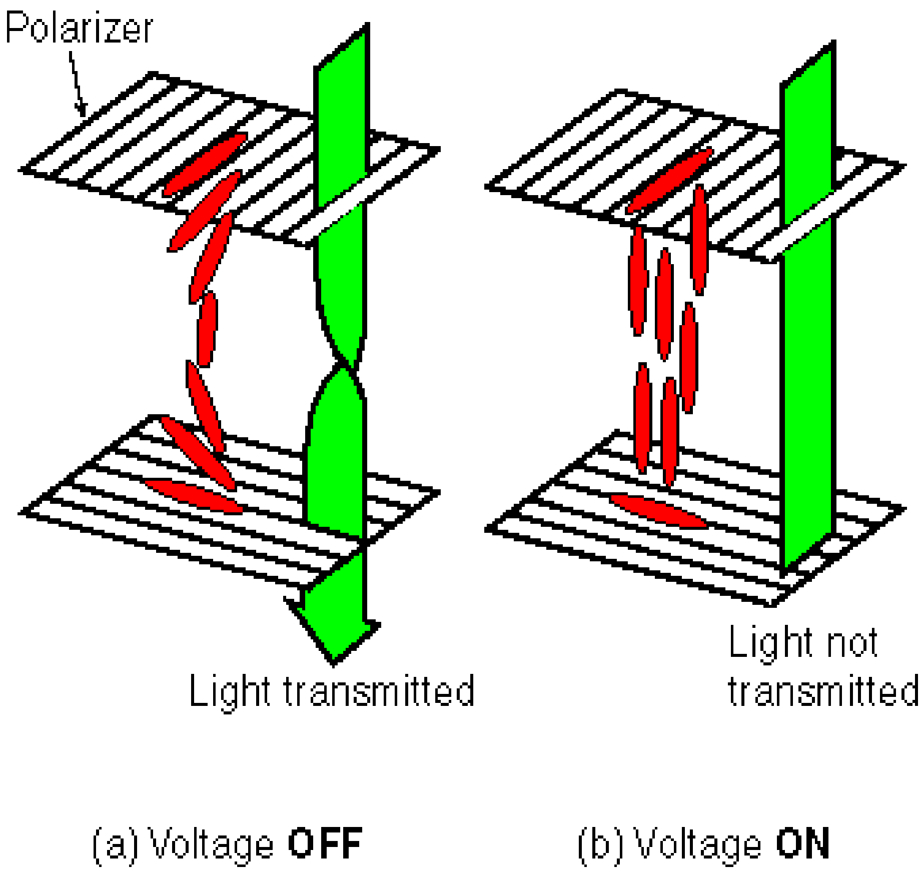


Display Applications

<u>Key properties:-</u>

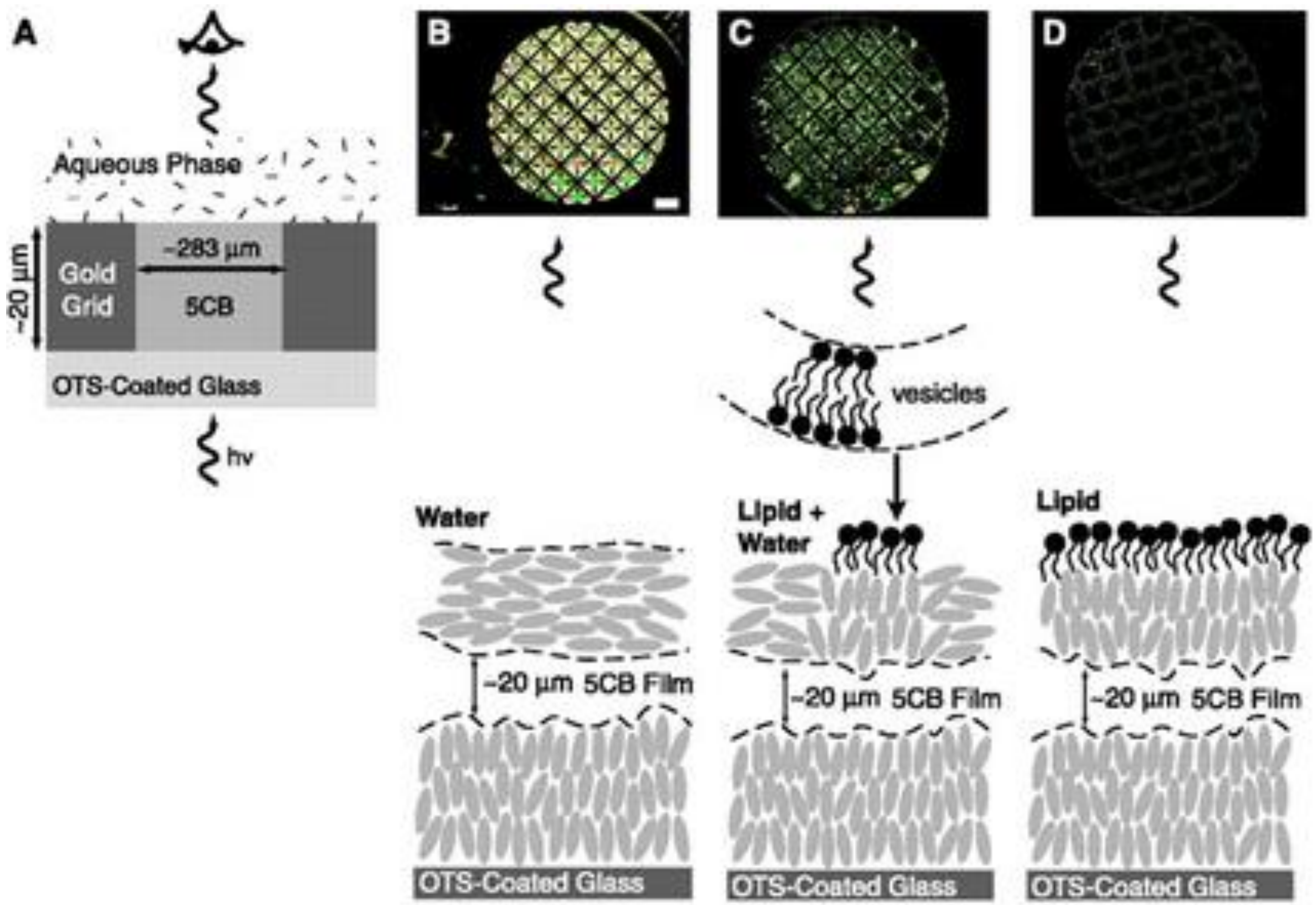
• Anisotropic birefringent fluids – strong coupling to incident light

• Sensitive to external electric and magnetic fields .



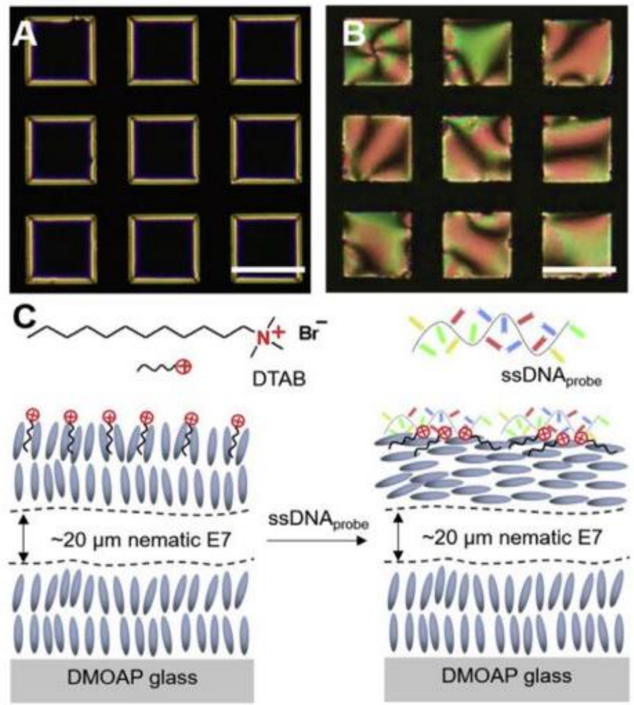
<u>Twisted Nematic Liquid Crystal Display</u> – a monostable liquid crystal display.

Bio-Applications in Sensors…



https://nlabbottcornell.weebly.com/research.html

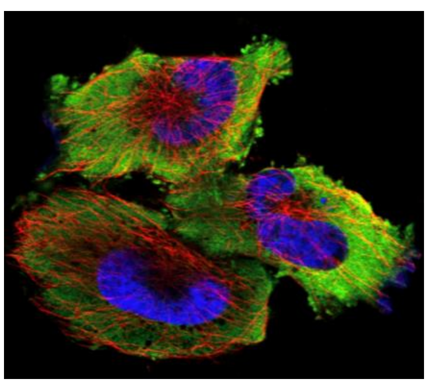
Virus Detection…



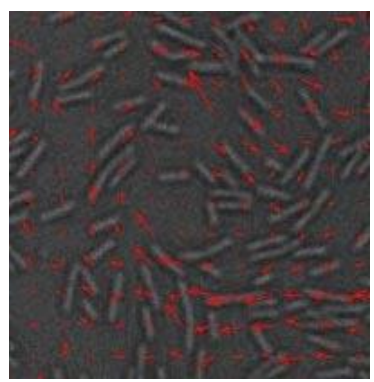
(i)

Rastogi et.al 2022, Journal of Molecular Liquids.

Active Matter…



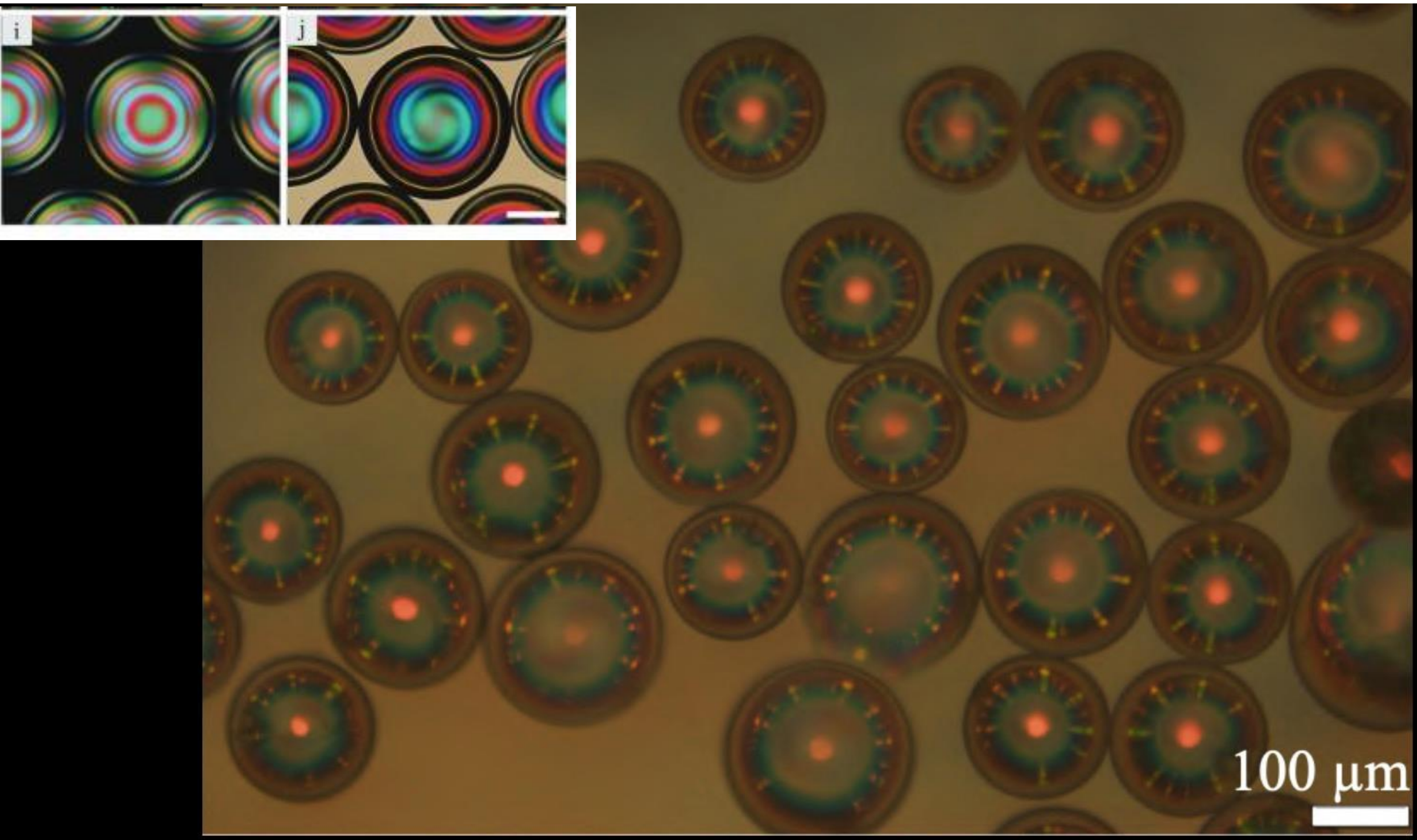
cytoplasm



bacteria

fish

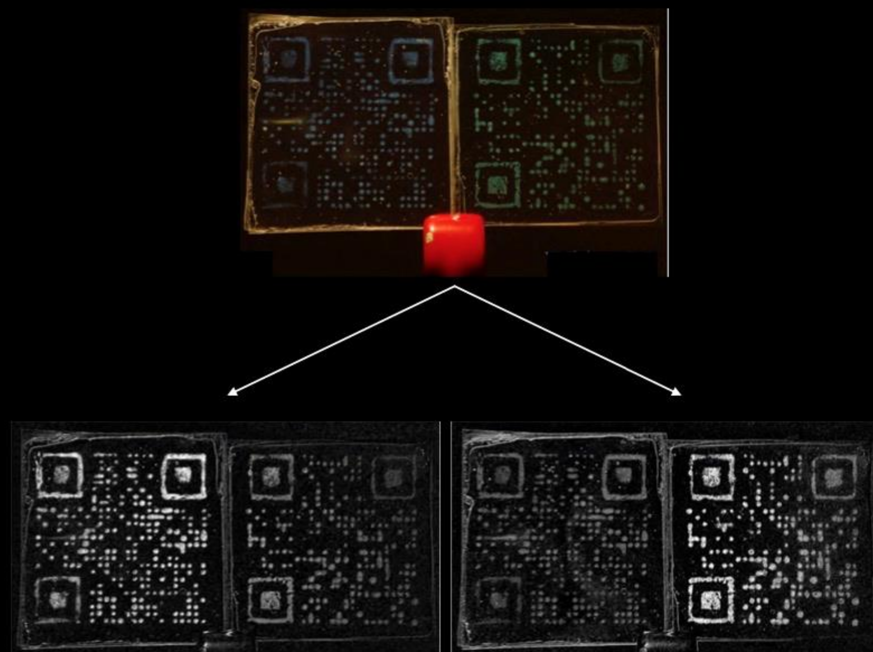
Applications in counterfeiting….



M. Schwartz et *al.*, *Multifunct. Mater.* **4**, 022002 (2021)

Applications in counterfeiting….

QR-codes made using CSR droplets



M. Schwartz et *al.*, *Multifunct. Mater.* **4**, 022002 (2021)

UK Liquid Crystal Research

The Proud Legacy of Strathclyde

- **Professor Frank Matthews Leslie** – moved to Strathclyde in 1968 and built the largest theory group for liquid crystals at Strathclyde

- Ericksen-Leslie theory of nematodynamics based on conservation of mass, energy, linear and angular momentum.

SOME CONSTITUTIVE EQUATIONS FOR ANISOTROPIC FLUIDS

By F. M. LESLIE
(*Department of Mathematics, University of Newcastle-upon-Tyne*)

[Received 3 December 1965. Revise 6 April 1966]

SUMMARY

The theory of a continuum with a director is considered and constitutive equations relevant to a class of anisotropic fluids discussed. Some exact solutions for simple shear, Poiseuille and Couette flow are given.

1. Introduction

RECENTLY Green and Rivlin (**1**) have developed a general theory of multipolar continuum mechanics which employs types of multipolar displacements. In a subsequent paper Green, Naghdi, and Rivlin (**2**)

Some Constitutive Equations for Liquid Crystals

F. M. LESLIE

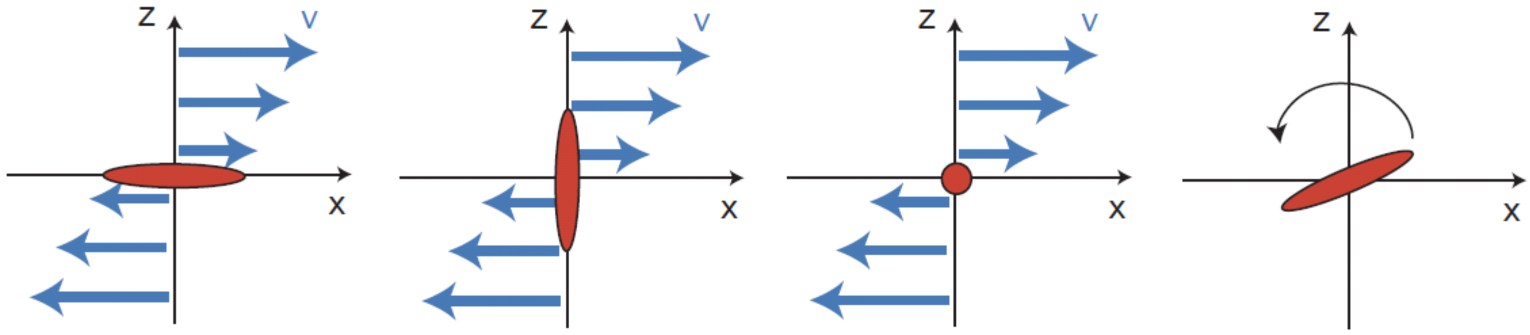
Communicated by J. L. ERICKSEN

1. Introduction

Since their discovery at the end of the last century, the materials called liquid crystals have aroused considerable interest. Briefly, liquid crystals are states of matter which are capable of flow, and in which the molecular arrangement gives rise to a preferred direction. BROWN & SHAW [1] discuss many of their properties in an extensive review. A number of attempts have been made to formulate continuum theories to describe properties of these peculiar liquids. The earliest seems to be OSEEN's static theory [2], which was later revised by FRANK [3]. More recently ERICKSEN [4] has reformulated these static theories. ANZELIUS [5] and OSEEN [6]

Courtesy: Professor Nigel Mottram

Frank worked out how the internal stress in a liquid crystal was related to the velocity gradients



(a) $\eta_1 = \frac{1}{2}(\alpha_3 + \alpha_4 + \alpha_6)$, (b) $\eta_2 = \frac{1}{2}(-\alpha_2 + \alpha_4 + \alpha_5)$, (c) $\eta_3 = \frac{1}{2}\alpha_4$, (d) $\gamma_1 = \alpha_3 - \alpha_2$

Each of these situations will have a different amount of friction

$$\tilde{t}_{ij} = \alpha_1 n_k n_p D_{kp} n_i n_j + \alpha_2 N_i n_j + \alpha_3 N_j n_i + \alpha_4 D_{ij} + \alpha_5 D_{ik} n_k n_j + \alpha_6 D_{jk} n_k n_i$$

and the α_i constants are now called the **Leslie viscosities**

Courtesy: Professor Nigel Mottram

- Ericksen-Leslie theory for nematic liquid crystals

Some Constitutive Equations for Liquid Crystals

F. M. Leslie

Communicated by J. L. Ericksen

- Leslie-Stewart-Nakagawa theory for smectic liquid crystals (1991)

A Continuum Theory for Smectic C Liquid Crystals

F. M. LESLIE

Mathematics Department, University of Strathclyde, Livingstone Tower, 26 Richmond Street, Glasgow G1 1XH, Scotland

and

I. W. STEWART

Department of Theoretical Mechanics, University of Nottingham, University Park, Nottingham NG7 2RD, England

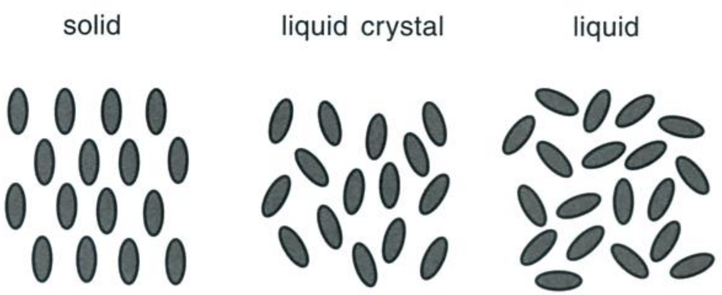
and

M. NAKAGAWA

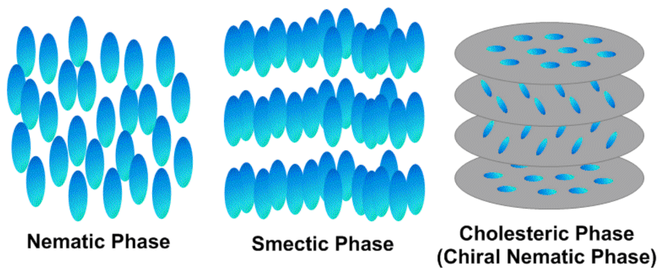
Department of Electrical Engineering, Faculty of Engineering, Nagaoka University of Technology, Kamitomioka 1603-1, Nagaoka, Niigata 940-21, Japan

UK Liquid Crystal Research

- ✓ University of Strathclyde (large theory group)
- ✓ University of Glasgow; University of Aberdeen
- ✓ Scottish Microelectronics Centre, University of Edinburgh
- ✓ Heriot Watt and Dundee
- ✓ Two national Strathclyde-led networks funded by the Royal Society of Edinburgh and the Isaac Newton Institute
- ✓ Strathclyde-led Liquid Crystal Reading Group
- ✓ Other UK centres of excellence – Leeds, Birmingham, Durham, Oxford, Bristol, Cambridge, Southampton
- ✓ Premier British Liquid Crystal Society



(Felix, et al., 2015)



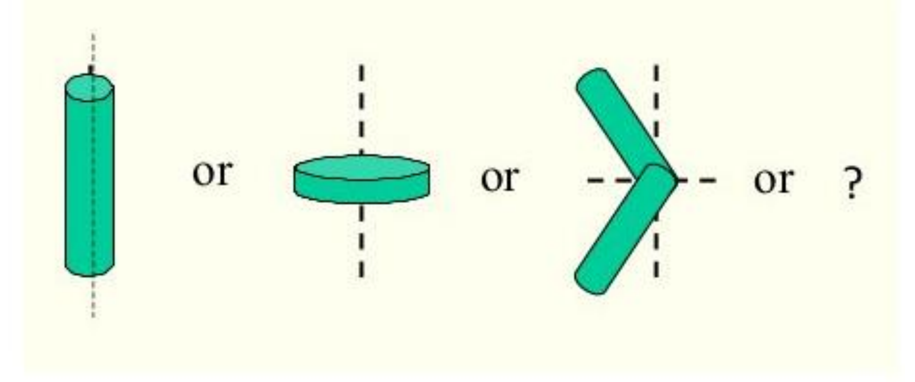
(tokyo chemical industry)

Liquid Crystals are a fascinating playground for mechanics, geometry, modelling, stochastics and analysis to meet physics and real-life applications.

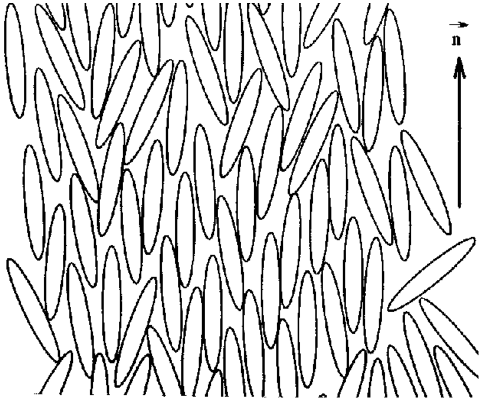
- Real opportunity for new mathematics-driven approaches to new materials, optimal design, optimal performance and efficient methodologies.

Nematic Liquid Crystals

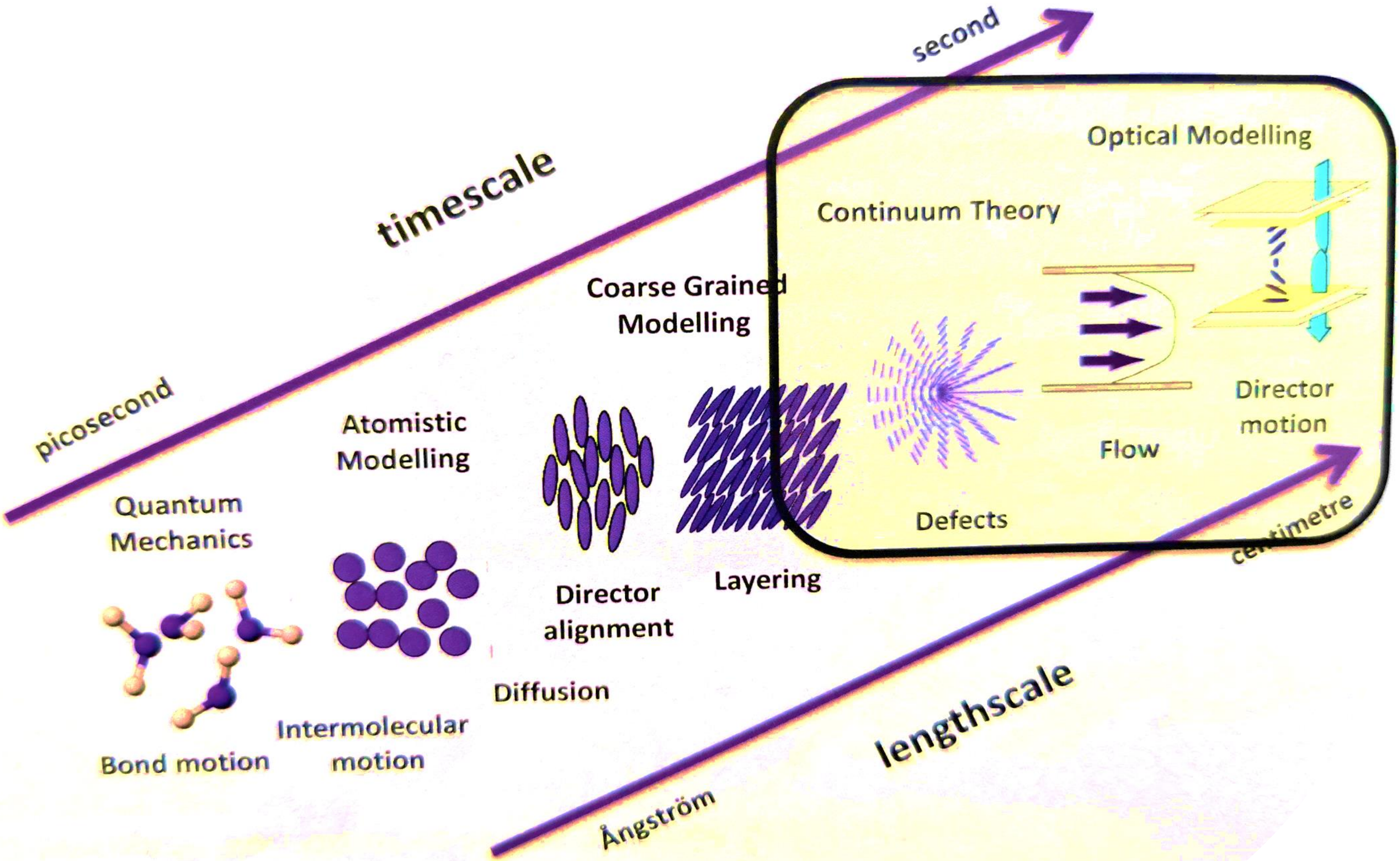
- ➢ Anisotropic rod-like molecules with directional properties



- ➢ Long-range orientational ordering: molecules line up with one another



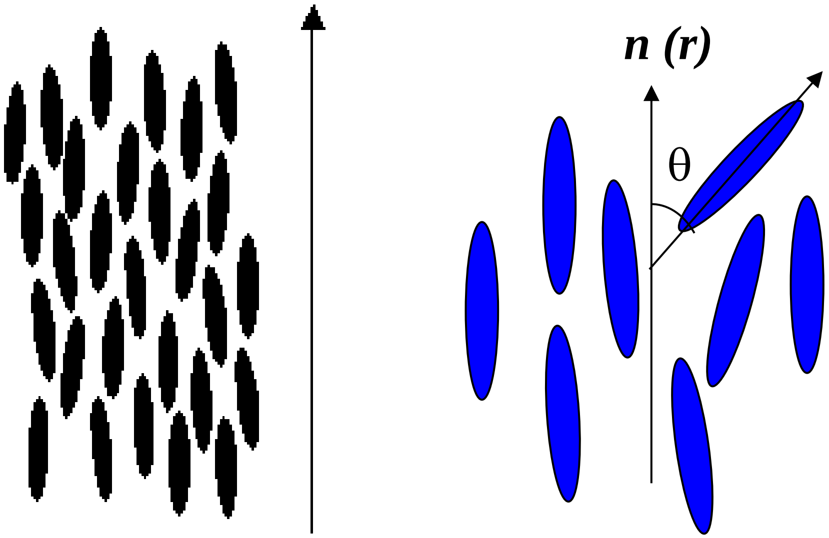
Mathematical modelling



(From Nigel Mottram’s lectures, BLCS Annual Training Workshop,

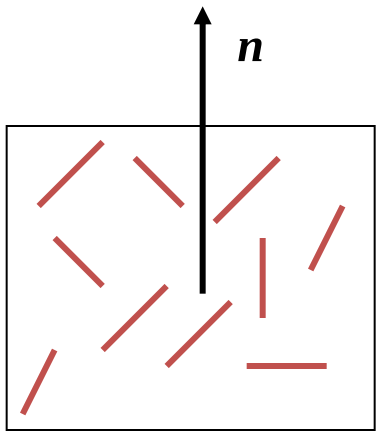
Important model parameters -

Nematic liquid crystals are anisotropic liquids with preferred directions of molecular alignment. The preferred alignment directions constitute the first set of important parameters.

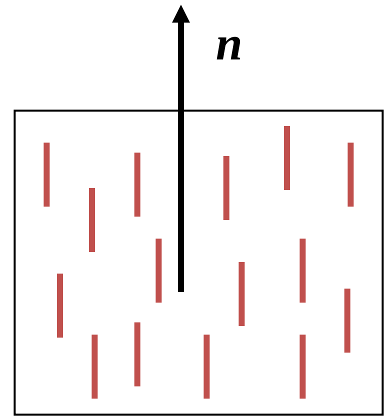


n (r) : preferred direction of orientation of the long molecular axes.

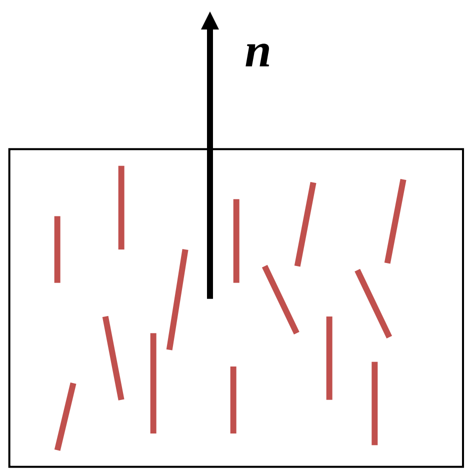
Scalar order parameter "S": a measure of the degree of alignment of the molecules.



S = 0; no alignment

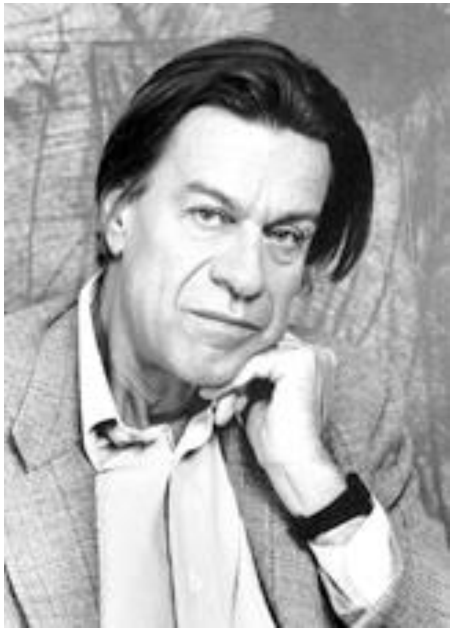


S = 1; perfect alignment



S ≈ 0.5; typical liquid crystal.

The Landau-de Gennes Theory



The Nobel Prize in Physics in 1991 was awarded to Pierre-Gilles de Gennes for "for discovering that methods developed for studying order phenomena in simple systems can be generalized to more complex forms of matter, in particular to liquid crystals and polymers“.

The Landau-de Gennes Theory

• General continuum theory that can account for most nematic phases and physically observable singularities.

•Define macroscopic order parameter that distinguishes nematic liquid crystals from conventional liquids, in terms of anisotropic macroscopic quantities such as the magnetic susceptibility and dielectric anisotropy.

• The **Q** – tensor order parameter is a symmetric, traceless 3×3 matrix.

$$Q = \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12} & Q_{22} & Q_{23} \\ Q_{13} & Q_{23} & -Q_{11} - Q_{22} \end{pmatrix}$$

- De Gennes' 1991 Nobel prize in Physics

  "for discovering that methods developed for studying order phenomena in simple systems can be generalized to more complex forms of matter, in particular to liquid crystals and polymers"

Five degrees of freedom.

The Landau-de Gennes Free Energy

The physically observable configurations are modelled by minimizers of the Landau-de Gennes liquid crystal energy functional subject to the imposed boundary conditions.

In the absence of any external fields and surface effects, the simplest form of the Landau-de Gennes energy is given by

$$I[Q] = \int_{\Omega} f_B(Q) + w(Q, \nabla Q)\, dV$$

The thermotropic potential : -

$$f_B(Q) = \frac{a}{2} \operatorname{tr} Q^2 - \frac{b}{3} \operatorname{tr} Q^3 + \frac{c}{4} \left(\operatorname{tr} Q^2 \right)^2 + C(a,b,c)$$

$$a = \alpha \left(T - T^* \right) \qquad \alpha, b, c, T^* > 0$$

- non-convex , non-negative potential with multiple critical points
- dictates preferred phase of liquid crystal – isotropic/ uniaxial/ biaxial?

The Landau-de Gennes Euler Lagrange Equations

The physically observable configurations correspond to minimizers of the Landau-de Gennes liquid crystal energy subject to the imposed boundary conditions.

The Euler-Lagrange equations in the one-constant case :

$$w(Q, \nabla Q) = L|\nabla Q|^2$$

$$\Delta Q_{ij} = \frac{1}{L}\left(AQ_{ij} - B\left(Q_{ip}Q_{pj} - \frac{1}{3}(trQ^2)\delta_{ij}\right) + C(trQ^2)Q_{ij}\right) \quad i,j = 1,2,3$$

$$\Delta Q_{ij} + \frac{L_2}{2}\left(Q_{ik,kj} + Q_{jk,ki} - \frac{2}{3}\delta_{ij}Q_{kl,kl}\right) = \frac{\lambda^2}{L}\left\{AQ_{ij} - B\left(Q_{ik}Q_{kj} - \frac{1}{3}\delta_{ij}\mathrm{tr}\mathbf{Q}^2\right) + CQ_{ij}\mathrm{tr}\mathbf{Q}^2\right\},$$

- ✓ System of nonlinear, coupled partial differential equations

What can mathematics tell us?

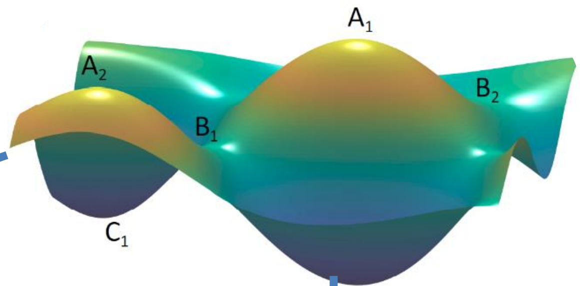
$$\Delta Q_{ij} = \frac{1}{L}\left(AQ_{ij} - B\left(Q_{ip}Q_{pj} - \frac{1}{3}(trQ^2)\delta_{ij}\right) + C(trQ^2)Q_{ij}\right) \quad i,j = 1,2,3$$

- ✓ Physically relevant/ observable states ⇔ Energy minimizers
- ✓ Energy minimizers ⇔ Classical Solutions of Euler-Lagrange Equations
- ✓ Asymptotic Analysis ⇒ Defect Sets
- ✓ Non- Energy Minimising Solutions ⇒ Switching Mechanisms

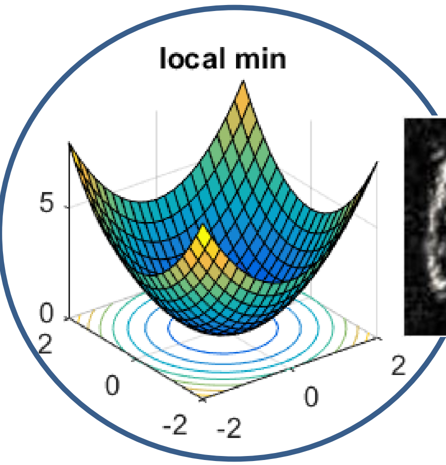
- ✓ **Modelling experiments and applications**
- ✓ **Predicting and Designing New Systems**
- ✓ **Control of Static and Dynamic Phenomena**

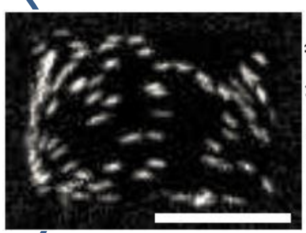
Some Concepts

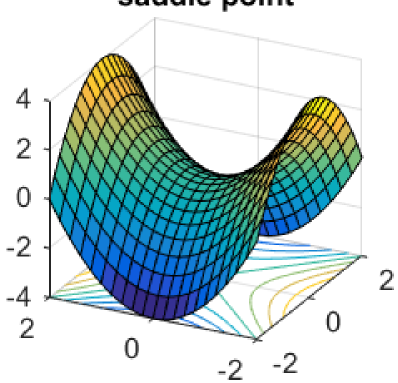
Energy landscape



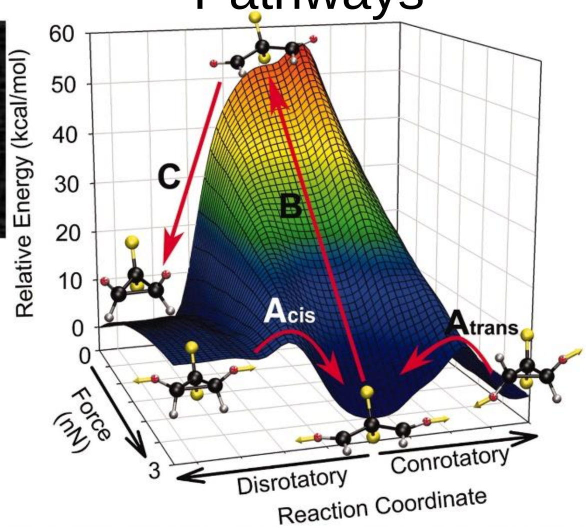
Stationary points/critical states





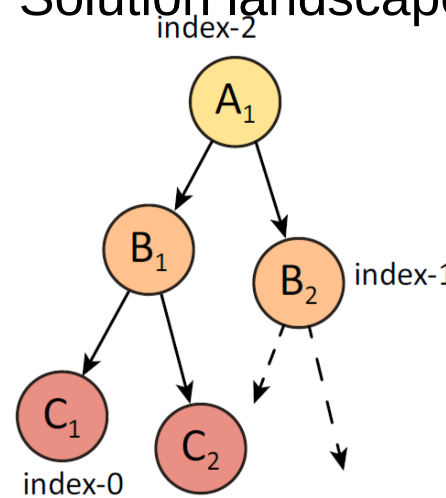


Pathways



Lenhardt, Jeremy M., et al., *Science* 329.5995 (2010): 1057-1060.

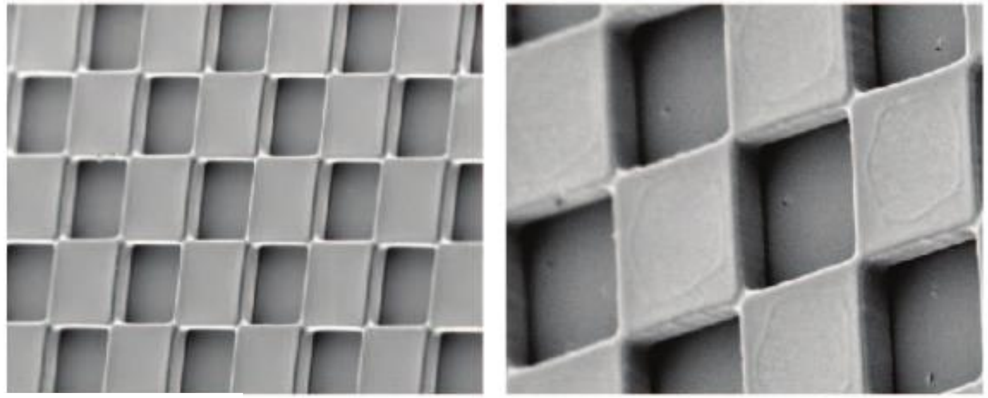
Solution landscape

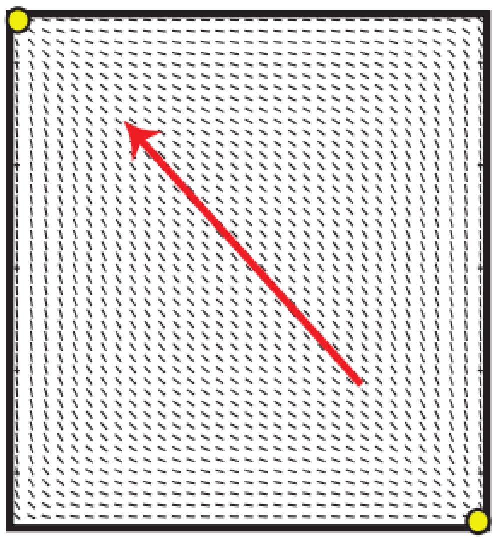


Solution landscape is a pathway map consisting of all stationary points and connections on energy landscape.

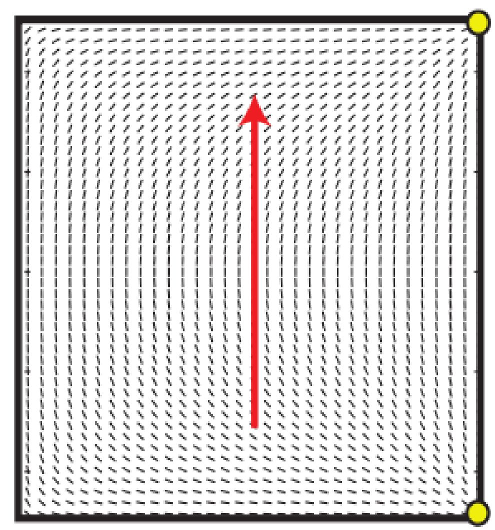
J. Yin, et. al., Physical Review Letters, 124:090601, 3 2020.

Physically Observable States in Applications





Tsakonas, Davidson, Brown, Mottram 2007



Reduced LdG Theory in Two Dimensions

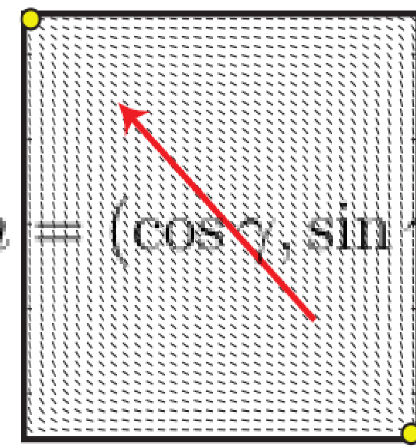
- Thin geometries – reduced LdG tensor with only two degrees of freedom

$$Q = \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12} & Q_{22} & Q_{23} \\ Q_{13} & Q_{23} & -Q_{11} - Q_{22} \end{pmatrix}$$

Fixed eigenvector with associated fixed eigenvalue

$$\downarrow$$

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{12} & -P_{11} \end{pmatrix}$$



$$n = (\cos\gamma, \sin\gamma)$$

$$P_{11} = s\cos 2\gamma \quad P_{12} = s\sin 2\gamma$$

Golovaty, Montero and Sternberg, 2015 Journal of Nonlinear Science, Volume 25, Issue 6,pp.1431-1451

Reduced Approaches contd.

- We can write $\mathbf{P}$ in terms of nematic scalar order parameter, s and direction $\mathbf{n} = (\cos\gamma, \sin\gamma)$,

$$\mathbf{P} = 2s\left(\mathbf{n} \otimes \mathbf{n} - \frac{1}{2}\mathbf{I_2}\right).$$

- The reduced Landau-de Gennes free energy is given by

$$E[P] = \int_{\Omega} \frac{\lambda^2}{L}\left(-\frac{A}{2}trP^2 + \frac{C}{4}(trP^2)^2\right) + \frac{1}{2}|\nabla P|^2 dA$$

- The corresponding Euler-Lagrange equations are

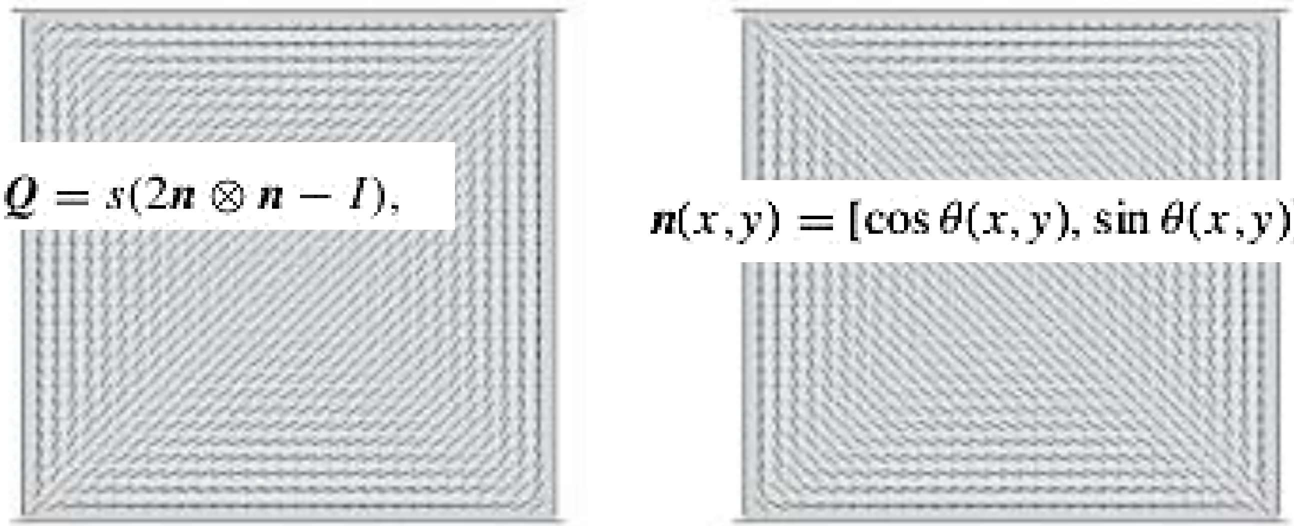
$$\Delta P_{11} = \frac{2C\lambda^2}{L}\left({P_{11}}^2 + {P_{12}}^2 - D(A,C)\right)P_{11},$$
$$\Delta P_{12} = \frac{2C\lambda^2}{L}\left({P_{11}}^2 + {P_{12}}^2 - D(A,C)\right)P_{12}.$$

See Maity, Majumdar and Nataraj, **IMA Journal of Numerical Analysis, 41 (2020), 1130–1163**

Two-Dimensional Approach continued…

$$Q = s(2\boldsymbol{n} \otimes \boldsymbol{n} - I),$$

$$\boldsymbol{n}(x, y) = [\cos\theta(x, y), \sin\theta(x, y)].$$



(a) **D1** (b) **D2**

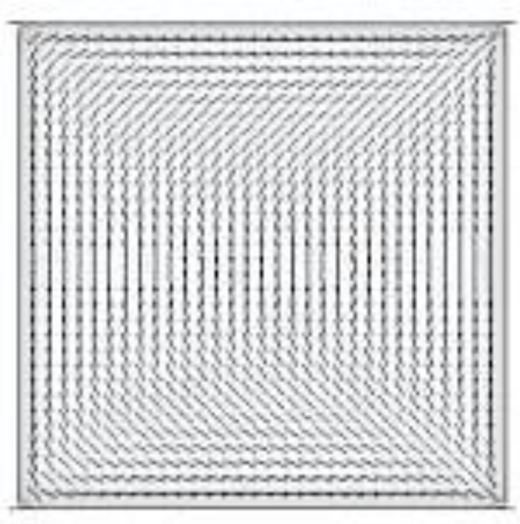
Plots of leading eigenvector with largest positive eigenvalue.

small values of $\epsilon \in (0.001, 0.01)$

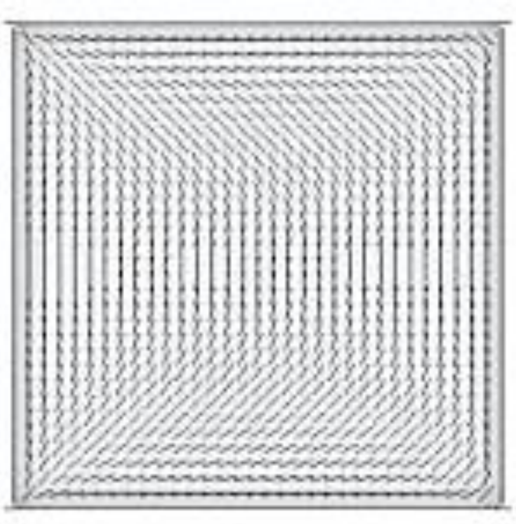
$$\mathcal{E}[Q] = \int_{\Omega} |\nabla Q_{11}|^2 + |\nabla Q_{12}|^2 + \frac{1}{\varepsilon^2}(Q_{11}^2 + Q_{12}^2 - 1)^2 dA$$

$$+ \int_{\partial\Omega} W|(Q_{11}, Q_{12}) - \mathbf{g}|^2 da, \qquad (47)$$

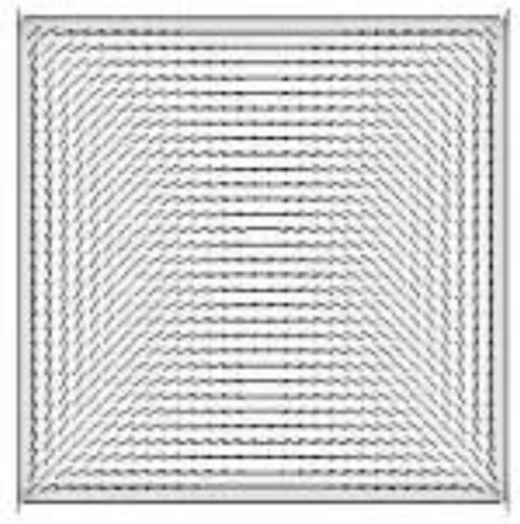
- Rotated Solutions



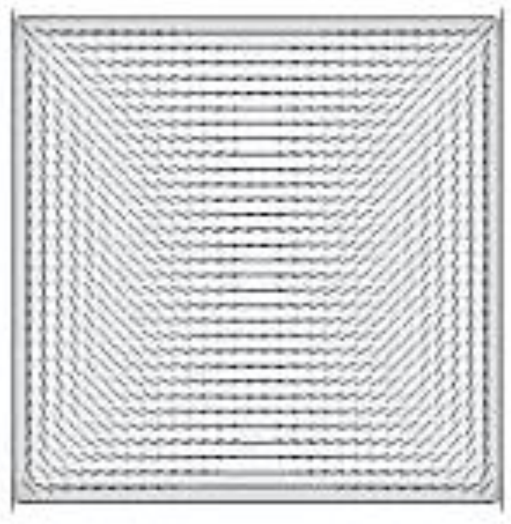
(c) R1



(d) R2

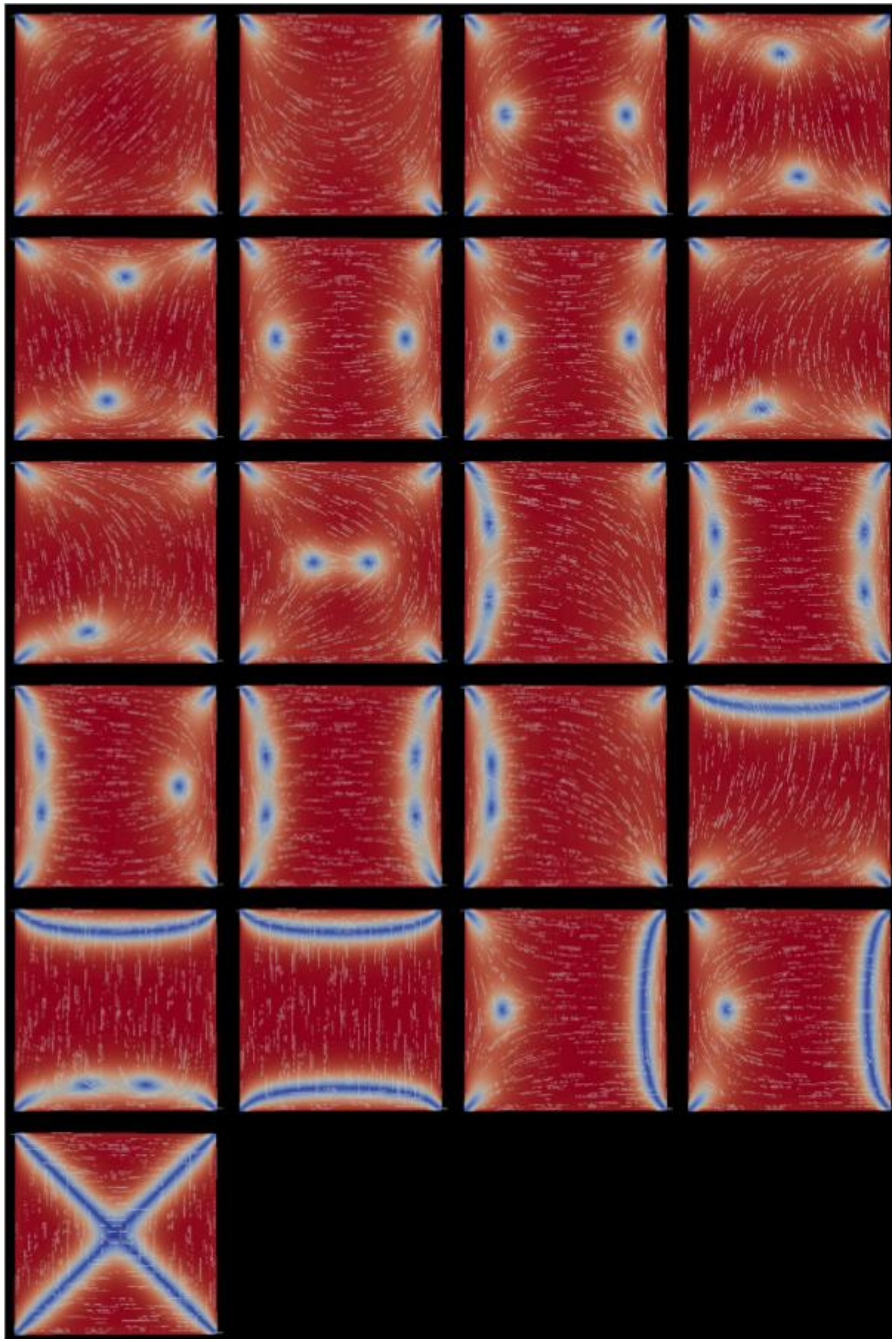


(e) R3



(f) R4

Plots of leading eigenvector with largest positive eigenvalue. Identify this eigenvector with the Oseen-Frank director.



Now make ϵ larger....

$$\mathcal{E}[Q] = \int_\Omega (|\nabla Q_{11}|^2 + |\nabla Q_{12}|^2) + \frac{1}{\varepsilon^2}\left(Q_{11}^2 + Q_{12}^2 - 1\right)^2 dA,$$

From molecular to continuum modelling of bistable liquid crystal devices
Martin Robinson, Chong Luo, Patrick E. Farrell, Radek Erban & Apala Majumdar
http://www.tandfonline.com/doi/abs/10.1080/02678292.2017.1290284

P. E. Farrell, Á. Birkisson, and S. W. Funke, "Deflation techniques for finding distinct solutions of nonlinear partial differential equations," *SIAM Journal on Scientific Computing*, vol. 37, p. A2026–A2045, 2015.

• Switching characteristics of the planar bistable liquid crystal device :

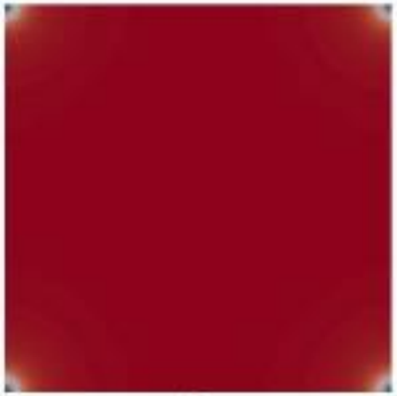
- dynamic model based on gradient flow approach

$$\frac{\partial Q_{11}}{\partial t} = \Delta Q_{11} - \frac{2}{\varepsilon^2}\left(Q_{11}^2 + Q_{12}^2 - 1\right) Q_{11} - \frac{1}{2}\mathrm{sgn}(C_0) E^2 \cos(2\theta_E) \quad \text{in } \Omega,$$

$$\frac{\partial Q_{12}}{\partial t} = \Delta Q_{12} - \frac{2}{\varepsilon^2}\left(Q_{11}^2 + Q_{12}^2 - 1\right) Q_{12} - \frac{1}{2}\mathrm{sgn}(C_0) E^2 \sin(2\theta_E) \quad \text{in } \Omega,$$

$$\frac{\partial Q_{11}}{\partial t} = -\frac{\partial Q_{11}}{\partial \boldsymbol{\nu}} - W(Q_{11} - g_1) \quad \text{on } \partial\Omega,$$

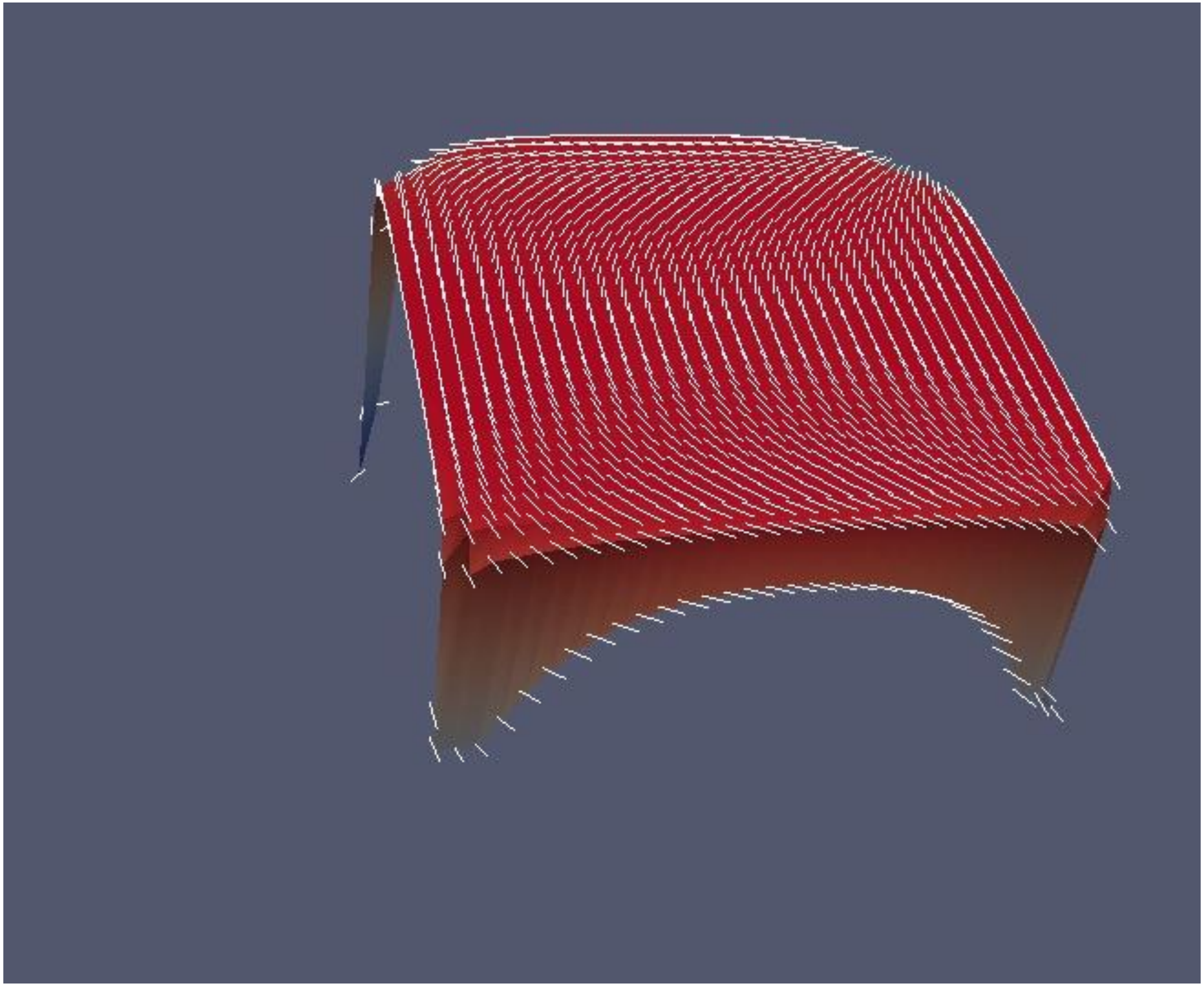
$$\frac{\partial Q_{12}}{\partial t} = -\frac{\partial Q_{12}}{\partial \boldsymbol{\nu}} - W(Q_{12} - g_2) \quad \text{on } \partial\Omega,$$

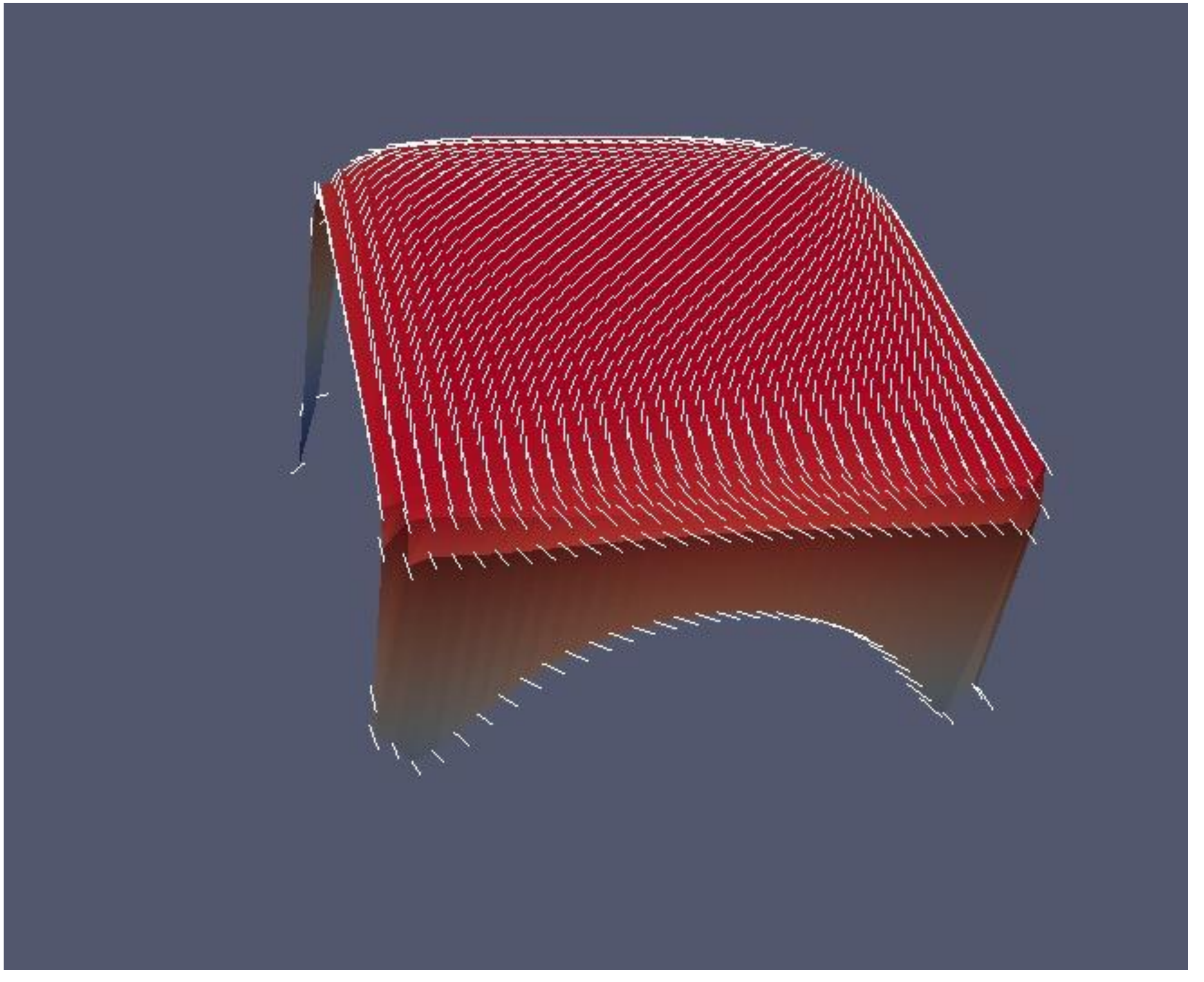


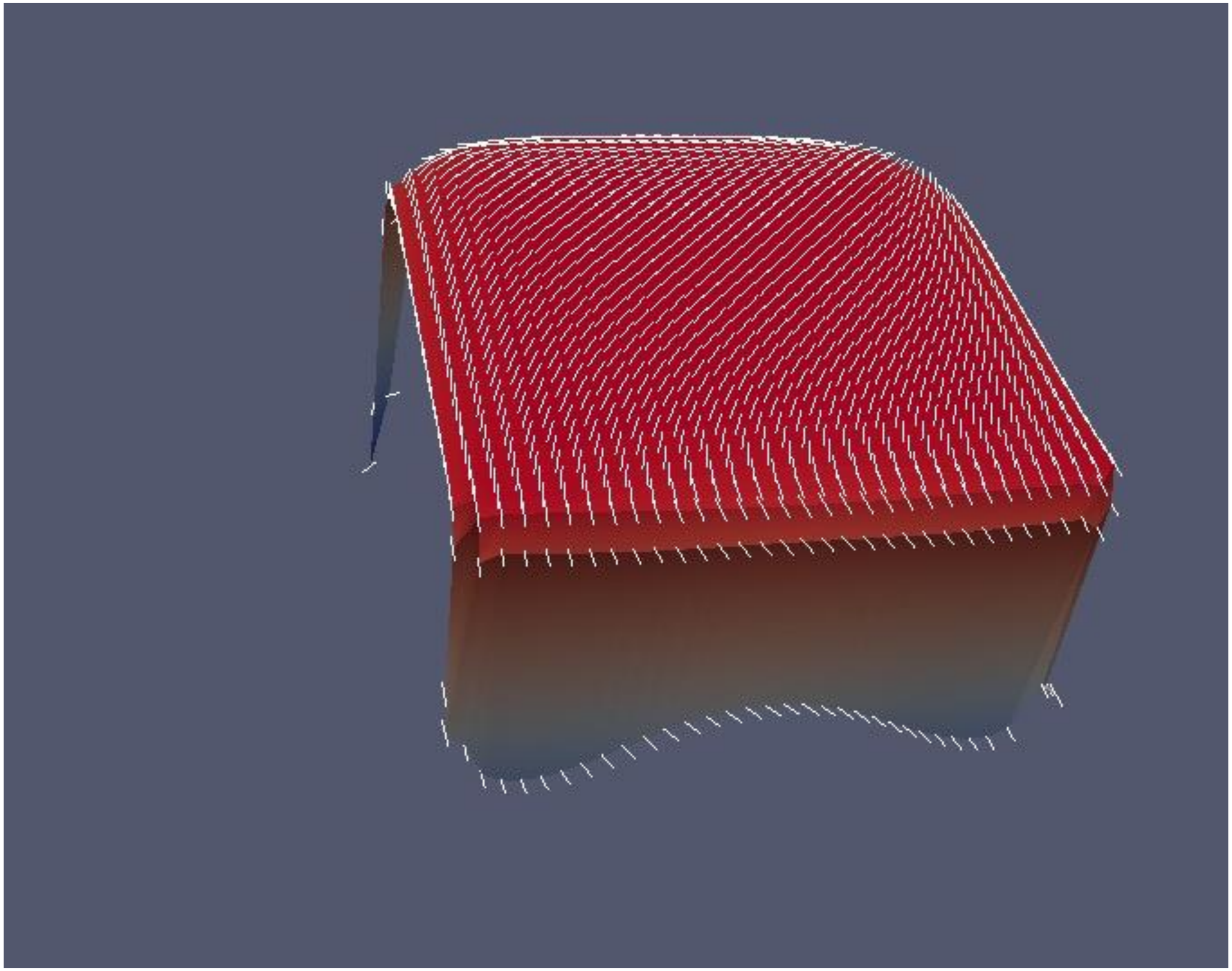
Order parameter profile inside domain – localized regions of reduced order near vertices.

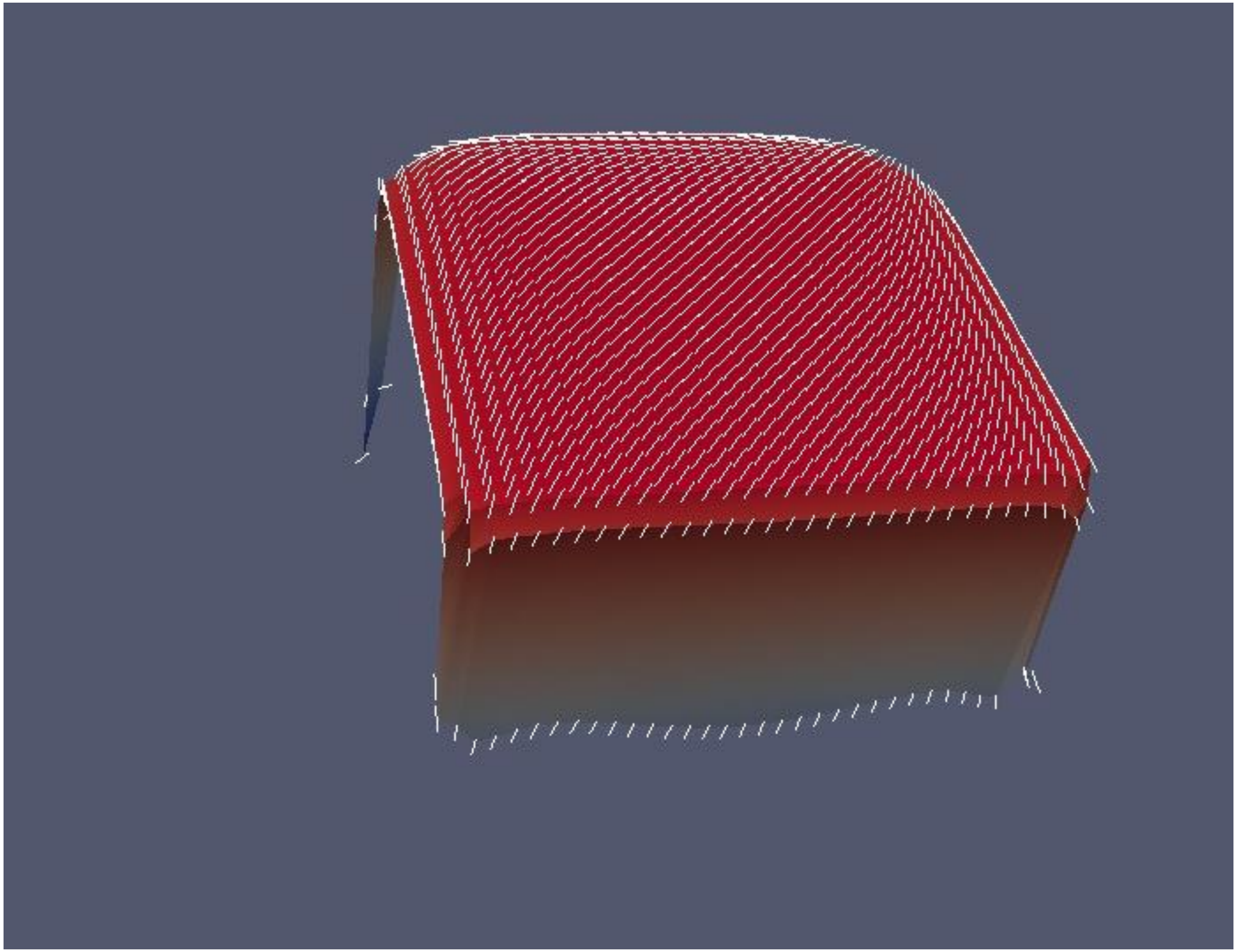
- Make the anchoring strength `W' different on different square edges (discussions with Professor Nigel Mottram)

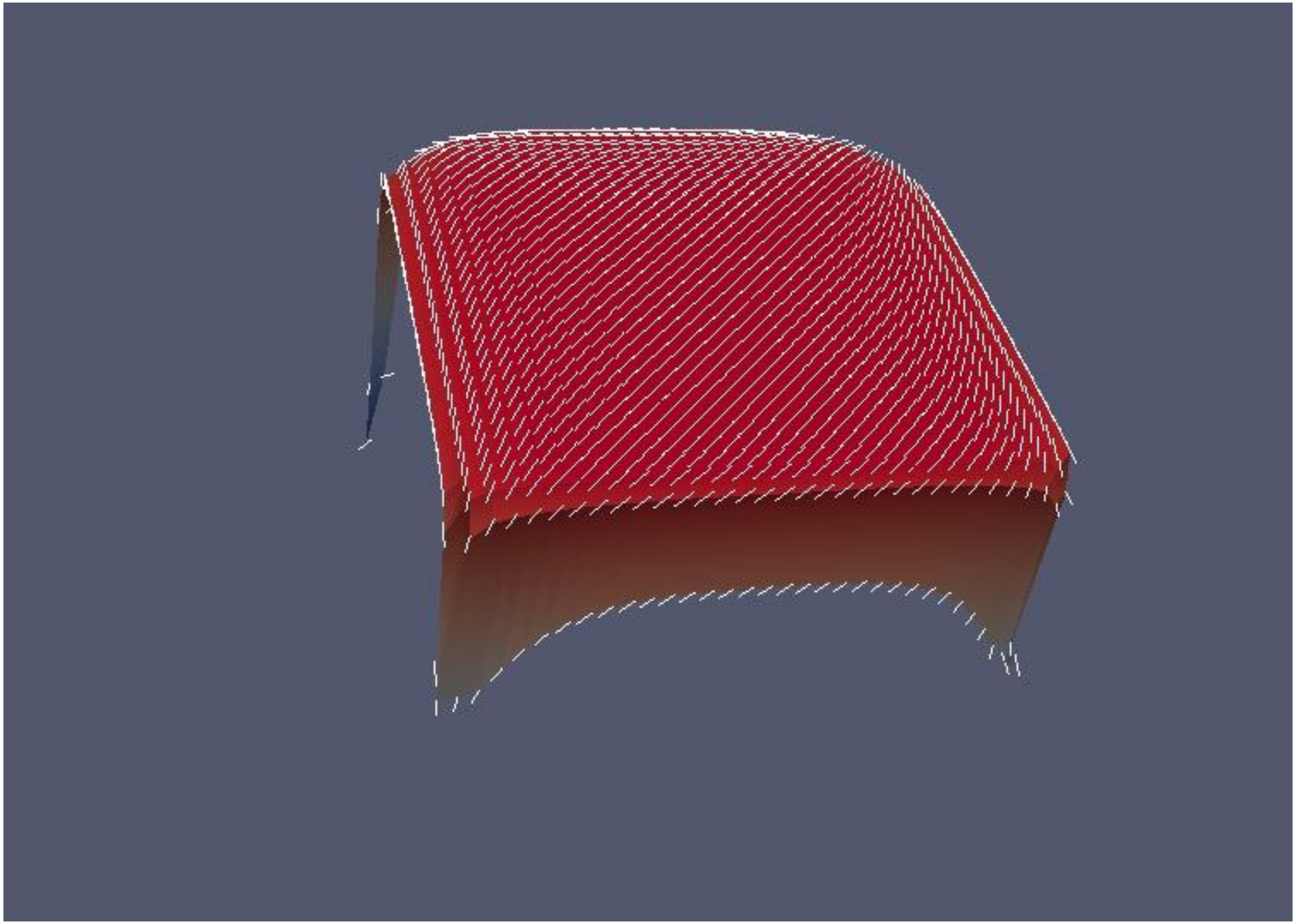
Rotated to Diagonal Switching

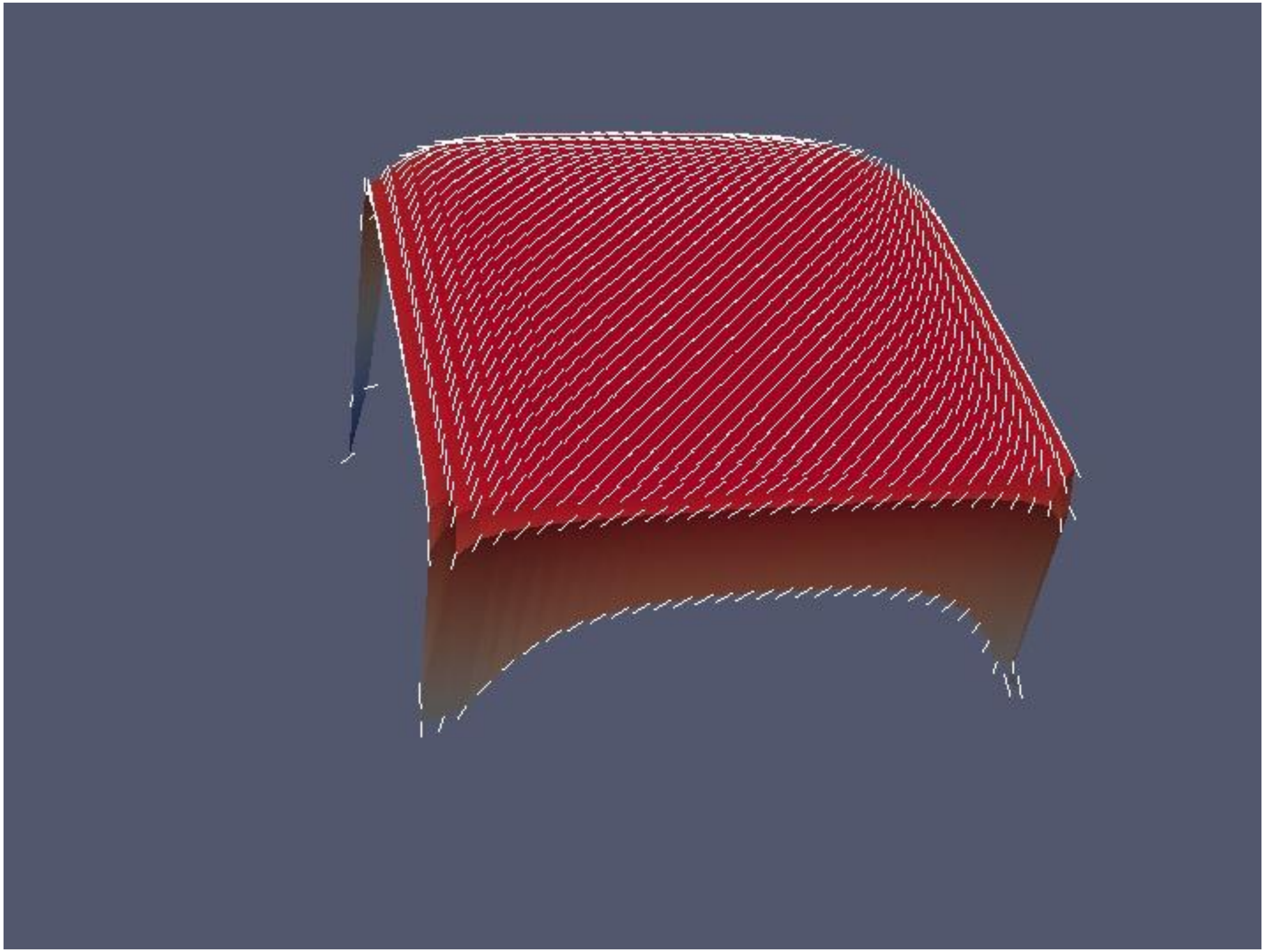




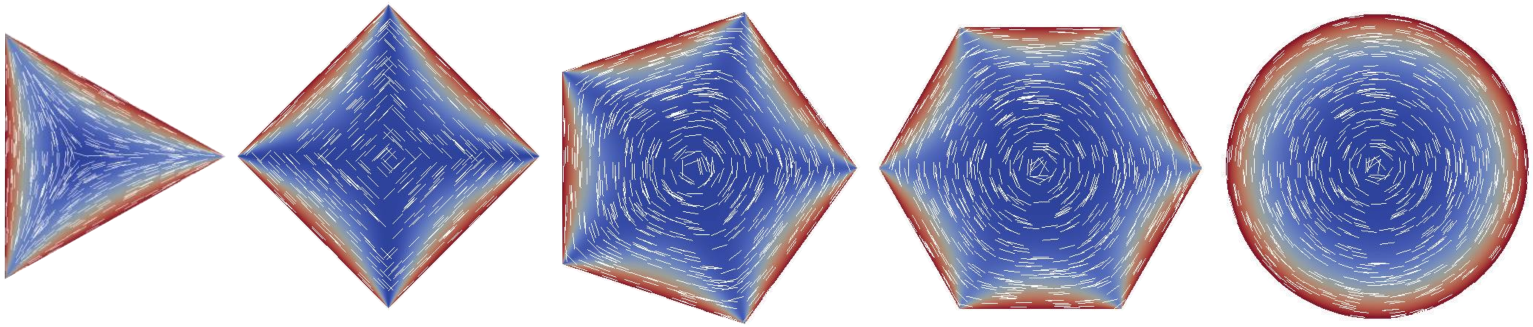




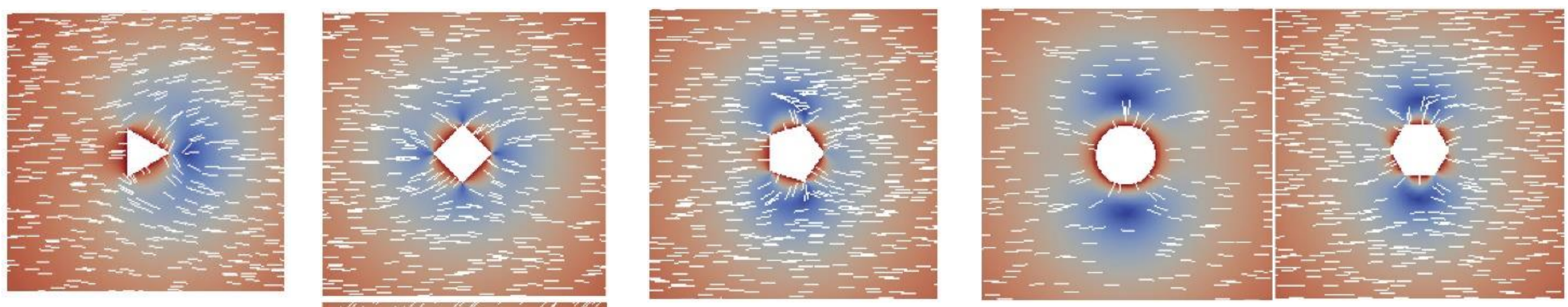


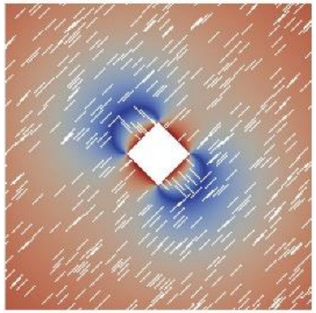


Generalizations to other 2D Polygons



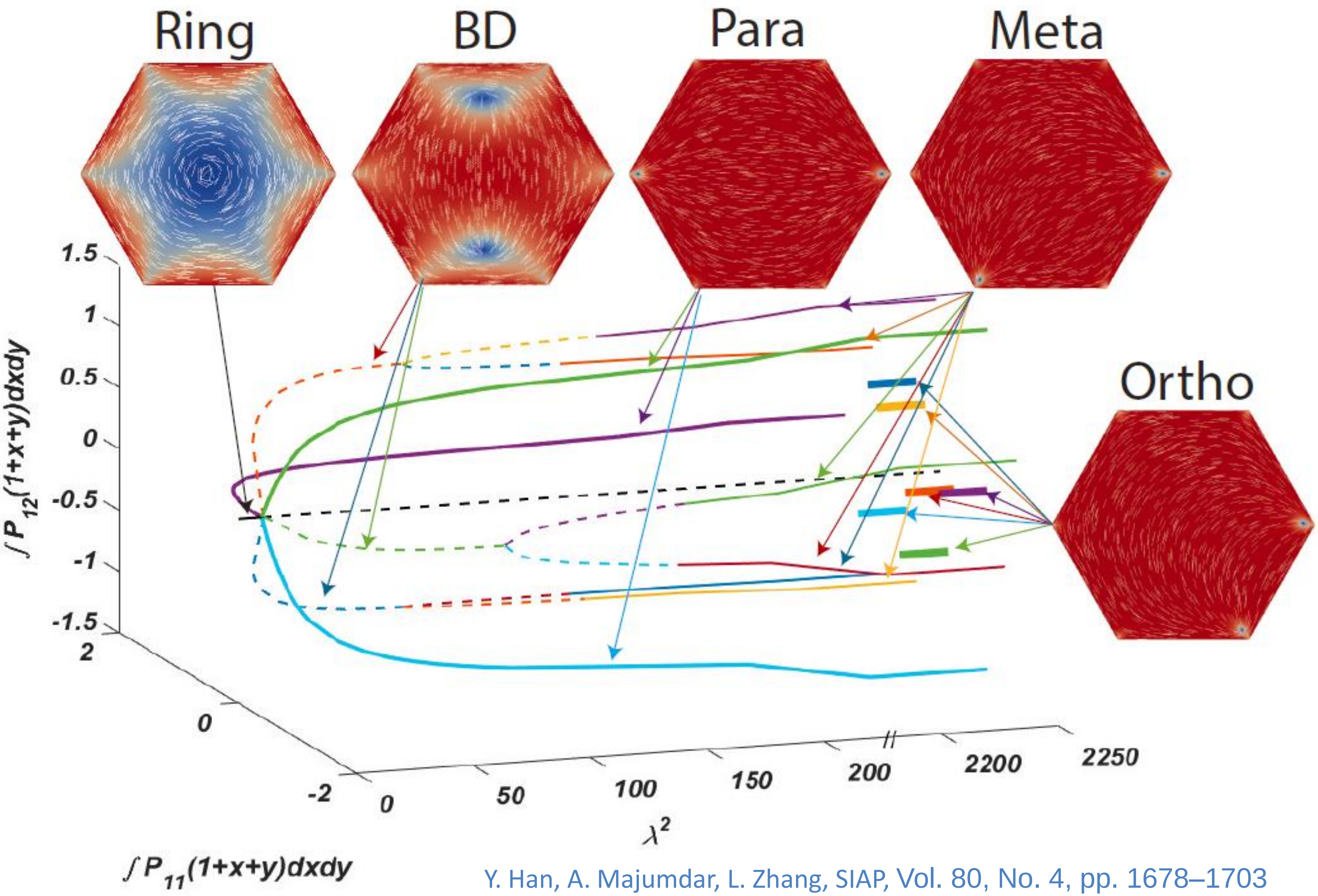
Y. Han, A. Majumdar, L. Zhang, SIAP, Vol. 80, No. 4, pp. 1678–1703





Y. Han, and A. Majumdar, *Journal of Nonlinear Science (2023)*.

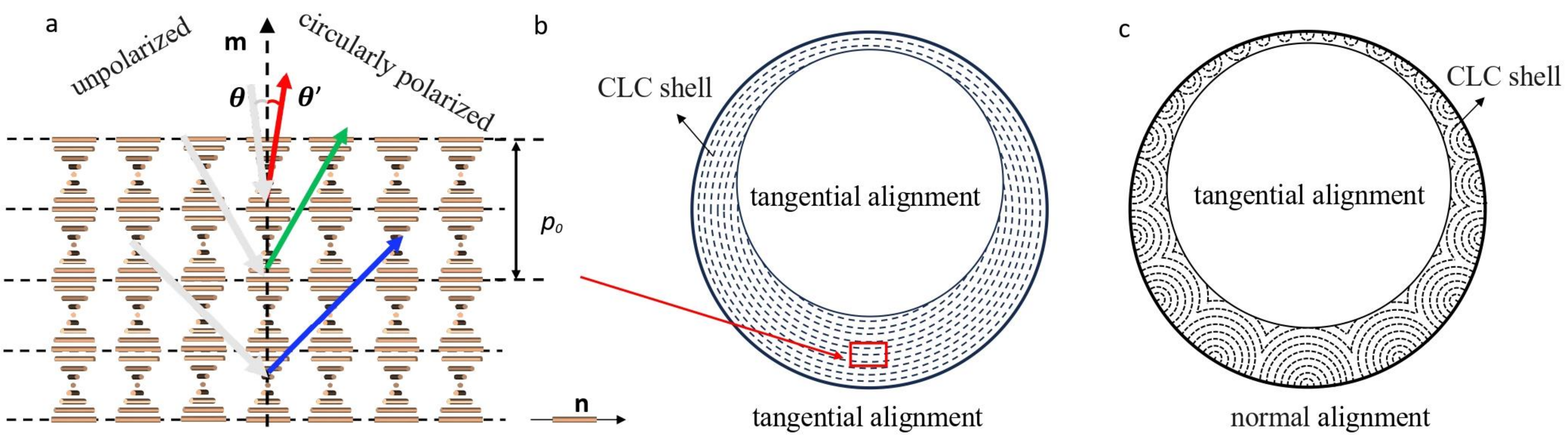
Bifurcation Diagram on a Hexagon as a function of Edge Length

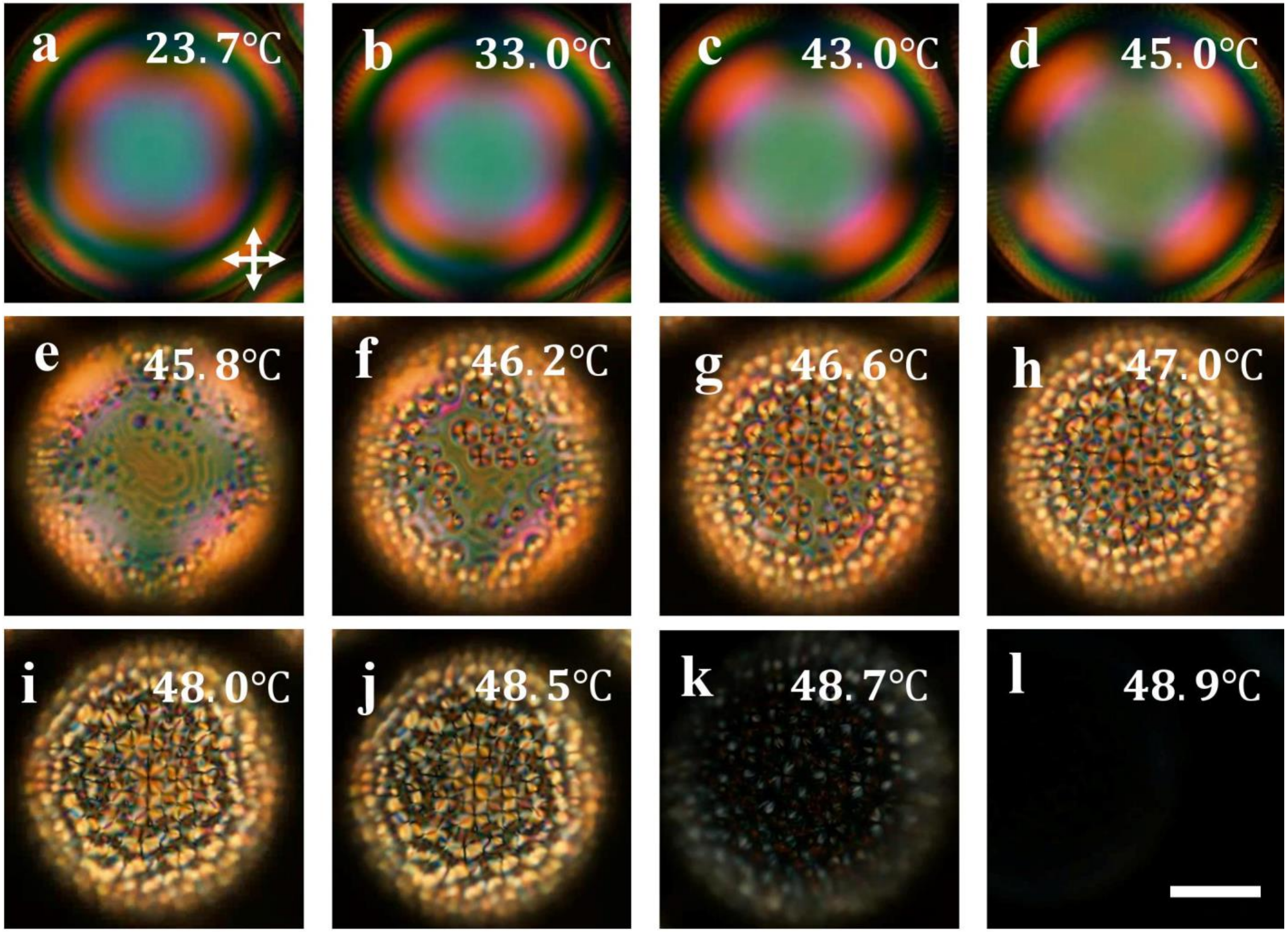


Y. Han, A. Majumdar, L. Zhang, SIAP, Vol. 80, No. 4, pp. 1678–1703

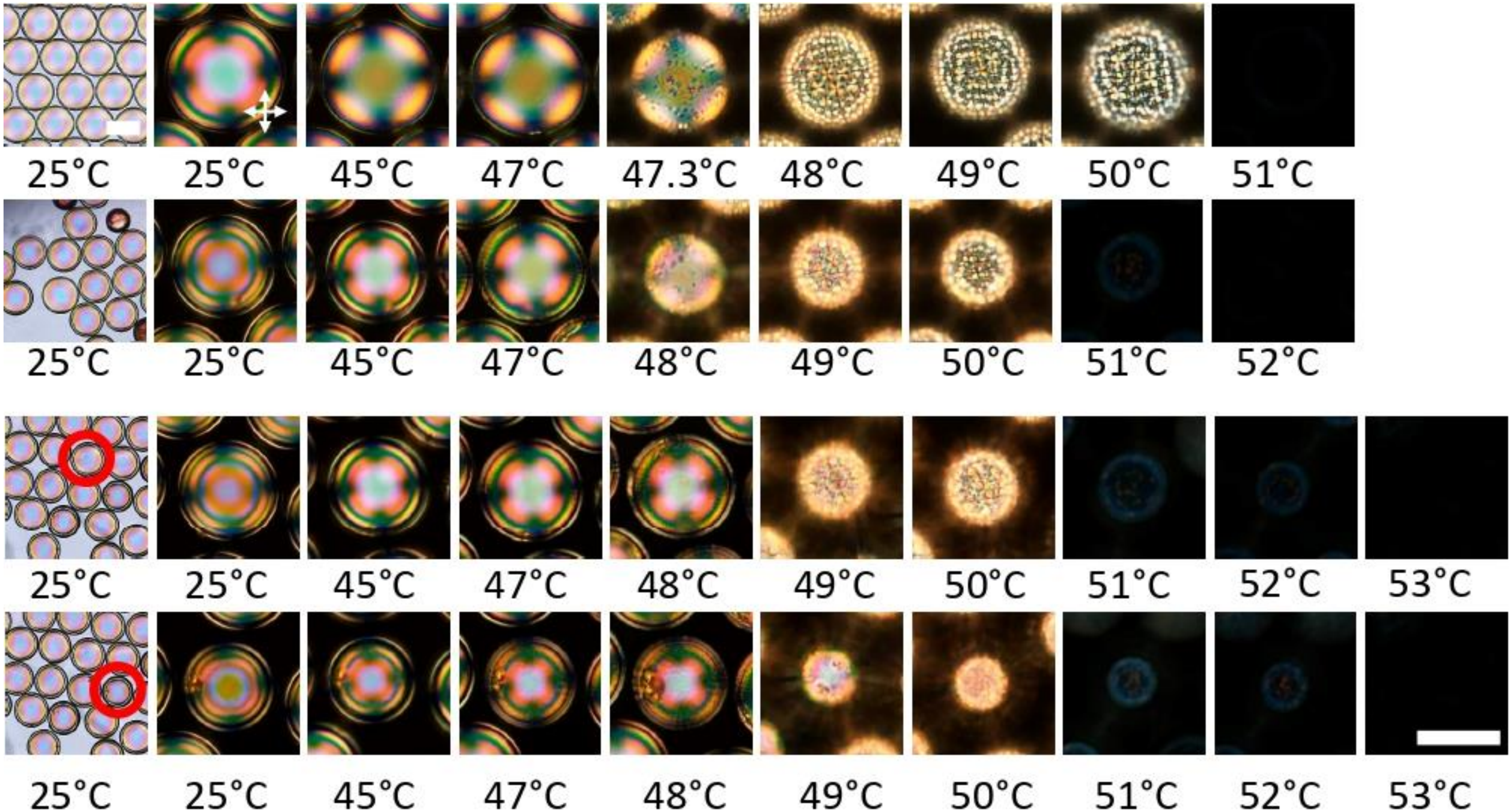
Heating Transitions for 3D Cholesteric Shells

✓ Xu Ma, Yucen Han, Yan-Song Zhang, Yong Geng, Apala Majumdar, and Jan P.F. Lagerwall, “Tunable templating of photonic microparticles via liquid crystal order-guided adsorption of amphiphilic polymers in emulsions” accepted for publication in Nature Communications, 2024





Heating transitions in CLC shells with PVA stabiliser; transitions from radial helical arrangements/planar director profiles to focal conic structures.



Impact of shell thickness on heating transitions in CLC shells with PVA stabiliser; Realignment temperature increases with shell thickness.

MODELLING DETAILS FOR CHOLESTERIC SHELLS

$$F(\mathbf{Q}) = \int_\Omega \frac{K_0}{2}(\nabla \cdot \mathbf{Q})^2 + \frac{K_1}{2}|\nabla \times \mathbf{Q} + 2q_0\mathbf{Q}|^2 + f_b(\mathbf{Q})dV,$$

$$f_b(\mathbf{Q}) := \frac{A}{2}\mathrm{tr}\mathbf{Q}^2 - \frac{B}{3}\mathrm{tr}\mathbf{Q}^3 + \frac{C}{4}(\mathrm{tr}\mathbf{Q}^2)^2.$$

$$\bar{\mathbf{x}} = \mathbf{x}/R_o,\ \bar{\mathbf{Q}} = \sqrt{\frac{27C^2}{2B^2}}\mathbf{Q},\ \bar{\mathcal{F}} = \frac{27C^3}{2B^4R_o^3}\mathcal{F}.$$

$$\mathcal{F}(\mathbf{Q}) = \int_{\boldsymbol{\Omega}} \left\{ \frac{\boldsymbol{\xi}_{\mathbf{R}}^{\mathbf{2}}}{\mathbf{2}}((\nabla \cdot \mathbf{Q})^{\mathbf{2}} + \eta\,|\nabla \times \mathbf{Q} + \mathbf{2}\sigma\mathbf{Q}|^{\mathbf{2}}) + \frac{\mathbf{t}}{\mathbf{2}}\mathrm{tr}\mathbf{Q}^{\mathbf{2}} - \sqrt{\mathbf{6}}\mathrm{tr}\mathbf{Q}^{\mathbf{3}} + \frac{\mathbf{1}}{\mathbf{2}}(\mathrm{tr}\mathbf{Q}^{\mathbf{2}})^{\mathbf{2}} \right\} \mathbf{dx}.$$

$$t = \frac{27AC}{B^2}, \xi_R = \sqrt{\frac{27CK_0}{B^2R_o^2}}, \eta = \frac{K_1}{K_0}, \sigma = q_0R_o$$

Boundary Conditions…

$$F_s = \frac{\omega}{2}\int_{\partial\Omega_k} |\mathbf{Q} - \mathbf{Q}^{\perp}|^2 dA,\ k = i\ or\ o, \qquad \mathbf{Q}^{\perp} = s_+(\mathbf{v}\otimes\mathbf{v} - \frac{\mathbf{I}}{3})$$

Normal boundary conditions.

$$F_s = \int_{\partial\Omega_k} \frac{\omega_1}{2}|\tilde{\mathbf{Q}} - \tilde{\mathbf{Q}}^{\parallel}|^2 + \frac{\omega_2}{2}(tr\tilde{\mathbf{Q}}^2 - s_+^2)^2 dA,\ k = i\ or\ o,$$

Tangent boundary conditions.

$$\tilde{\mathbf{Q}} = \mathbf{Q} + \frac{\mathbf{s}_+\mathbf{I}}{3},\ \tilde{\mathbf{Q}}^{\parallel} = \mathbf{P}\tilde{\mathbf{Q}}\mathbf{P},\ \mathbf{P} = \mathbf{I} - \mathbf{v}\otimes\mathbf{v},$$

Numerical Methods – Bispherical Polar Coordinates…

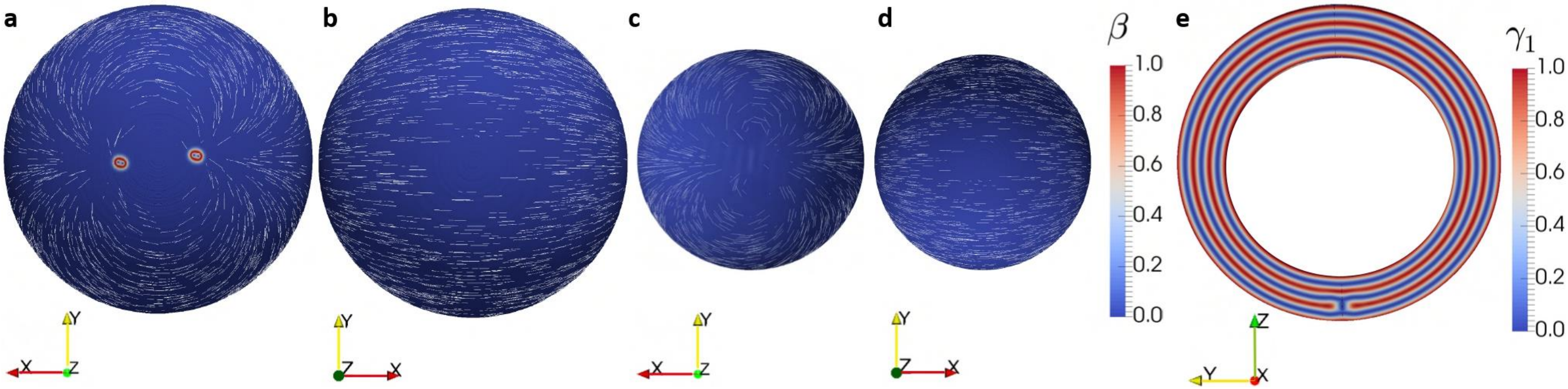
$$x = \frac{a\sin\mu\cos\psi}{\cosh\xi - \cos\mu},\ y = \frac{a\sin\mu\sin\psi}{\cosh\xi - \cos\mu},\ z = \frac{a\sinh\xi}{\cosh\xi - \cos\mu}$$

$$q_i(\zeta,\mu,\psi) = \sum_{l=0}^{L-1}\sum_{m=1-M}^{M-1}\sum_{n=|m|}^{N-1} A_{lnm}^{(i)} Z_{lnm}(\zeta,\mu,\psi),$$

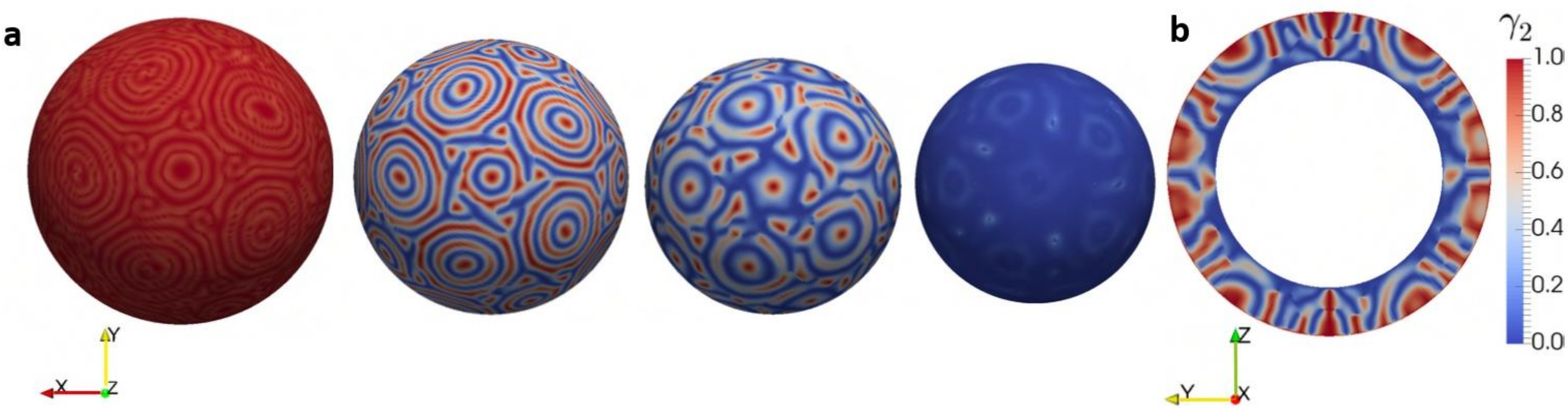
$$Z_{lnm}(\zeta,\mu,\psi) = P^l(\zeta)Y_{nm}(\mu,\psi),$$

$$Y_{nm} = P_n^{|m|}(\cos\mu)X_m(\psi),$$

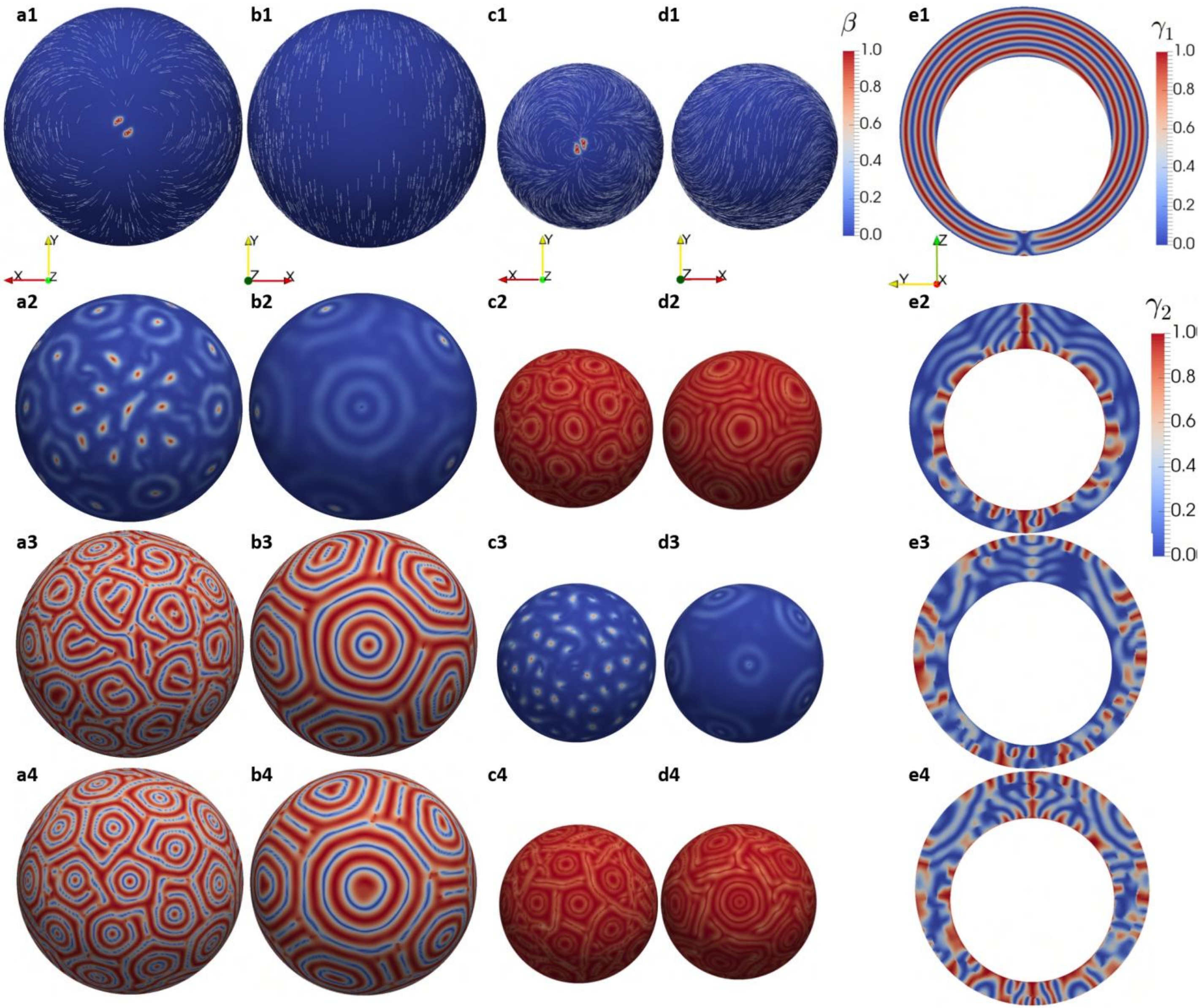
$$X_m(\psi) = \begin{cases}\cos m\psi, & \text{if } m \geq 0,\\ \sin|m|\psi, & \text{if } m < 0.\end{cases}$$



Tangential boundary conditions on the inner and outer surfaces.



Normal boundary conditions on the outer surface; tangential on the inner surface.



Ma, Han, Zhang, Gong, Majumdar and Lagerwall 2024 Tunable templating of photonic microparticles via liquid crystal order-guided adsorption of amphiphilic polymers in emulsions. Nature Communications.

Is Mathematics for You?

Absolutely yes – if you want it to be!

David Hilbert: “Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country”

Strathclyde Continuum Mechanics and Industrial Mathematics Group

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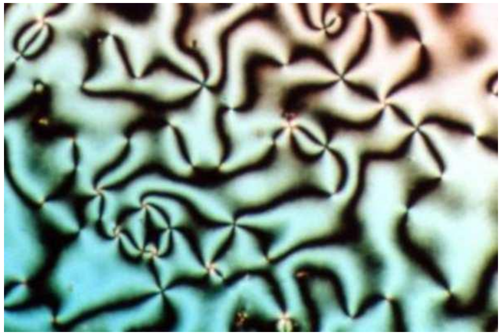
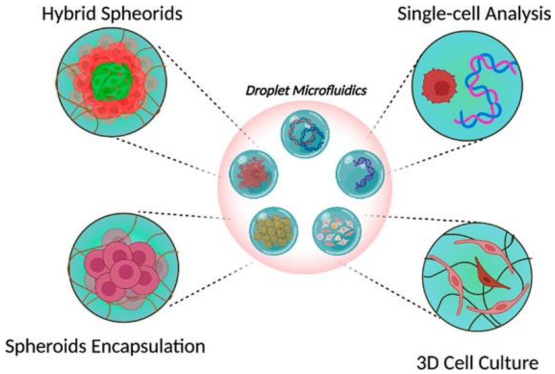
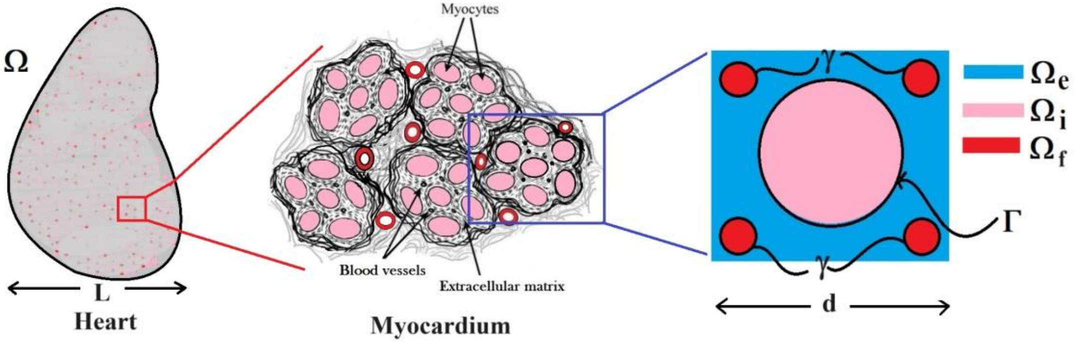
- Debasish Das
- Geoff McKay
- Apala Majumdar (Co-lead)
- Laura Miller
- Mikhail Osipov
- David Pritchard
- Alexander Wray (Co-lead)

The CMIM Group

- CMIM is an internationally leading research group in applied mathematics, with long-standing expertise in applying mathematics to real-world challenges and industrial problems, with a global reputation in the mathematics of liquid crystals, fluid dynamics (including complex fluids and droplet dynamics), polymer physics, poroelasticity and more.
- Come and speak to us about our research and work with us!!







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THANK YOU FOR YOUR ATTENTION!