

Maximising temporal reachability in trees

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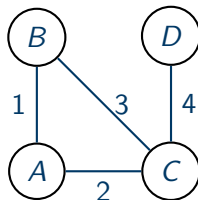
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Maximising spread through a network has uses in spreading information or resources.

Temporal Graphs

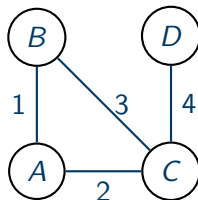
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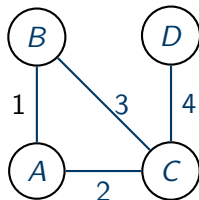
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A **strict temporal path** is a path of edges e_0, \dots, e_k such that each e_i is assigned a time by \mathcal{T} where $t(e_{i-1}) < t(e_i)$ for $1 \leq i \leq k$.



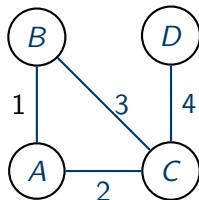
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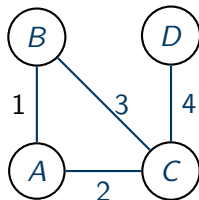
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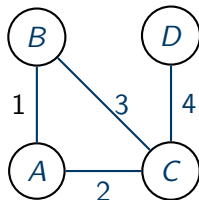


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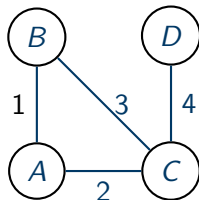
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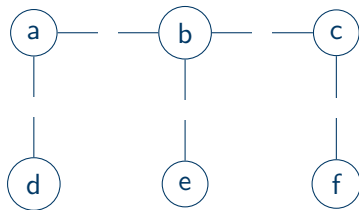
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The **minimum temporal reachability** of a graph G is the minimum cardinality of the temporal reachability sets of the vertices in G .



An Application

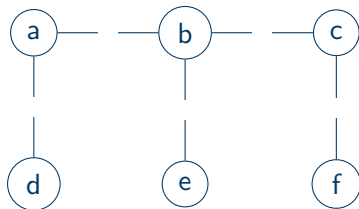
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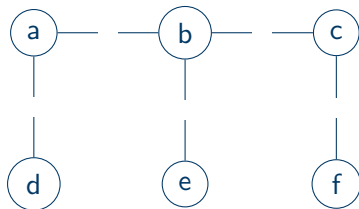


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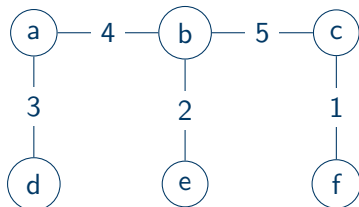


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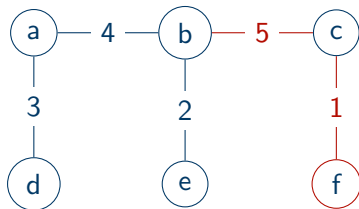


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- If so, which properties does the underlying graph need to have?
- Can we assign timesteps to edges such that the temporal reachability of every vertex falls within a given interval?
- Suppose we are performing an operation on our graph and we know some of the timesteps in the graph are wrong, how close can we get to an optimal solution?

Thanks for listening! Any questions?