Uncovering Product Cannibalisation

using Multivariate Hawkes Processes

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PiFORUM, September 2022

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Research Goal

We aim to quantify product cannibalisation for existing products in an apparel wholesale data set.

> *Product cannibalisation refers to the decrease in sales of one product due to (the introduction of) a closely related product. [Copulsky, 1976]*

The current focus is on **inference** to detect and understand product cannibalisation for products that already have a sales history.

Very important from a business perspective!

- Find a model
- Implement that model
- Fit that model on real data

Find A Model

Data: event times (plus additional covariates)
Let $N(t)$ be the number of observed events from 0 to t.
Homogeneous (λ constant) Poisson point process:

$$\mathbb{P}[N(t) = N] = \frac{(\lambda t)^N}{N!} e^{-\lambda t} \tag{1}$$

$$\mathbb{E}[N(t)] = \lambda t \tag{2}$$

Inhomogeneous ($\lambda(t)$ variable) Poisson point process:

$$p(t_1 \ldots t_N) = \prod_{i=1}^{N} \lambda(t_i)\, e^{-\int_0^\infty \lambda(z)\, dz} \tag{3}$$

$$\mathbb{E}[N(t)] = \int_0^t \lambda(z)\, dz \tag{4}$$

[Daley and Vere-Jones, 2003]

Hawkes processes [Hawkes, 1971] are a class of point processes that are used to model event data when the events can occur in clusters or bursts. They are defined on the interval $[0, T]$ with **conditional intensity function**:

$$\lambda(t) = \mu(t) + \sum_{i:t>t_i} K\, g(t - t_i) \quad (5)$$

Here, $\mu(t)$ can capture seasonality and underlying trends and we use $g(t - t_i) = \beta e^{-\beta(t-t_i)}$ for the self-excitement.

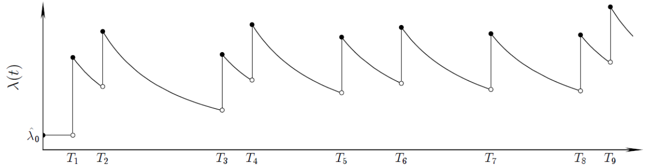

Figure 1: Intensity function with self exciting kernel [Rizoiu et al., 2017]

Assume that there are M dimensions with data $Y_1 = (t_{1\,1} \dots t_{1\,N_1}) \dots Y_M = (t_{M\,1} \dots t_{M\,N_M})$. At time t the intensity in dimension i is defined as the sum of the background rate $\mu_i(t)$ and contributions from all dimensions:

$$\lambda_i(t) = \left[\mu_i(t) + \sum_{j=1}^{M} \sum_{l: t > t_{j\,l}} K_{ji}\, g_{ji}(t - t_{j\,l}) \right]_+ \tag{6}$$

We assume the following form for the influence for all i, j: $g_{ij}(x) > 0$ for $x > 0$ and $\int_0^\infty g_{ij}(x)\, dx = 1$. Here, each $K_{ij} < 1$, and we write them as matrix $\mathbf{K} = \{K_{ij}\}$ where $i, j = 1 \dots M$.

We use a **multivariate** Hawkes Process where each dimension represents one product. This allows us to estimate the 'influence' K_{ij} from an event (sale) of one product i onto each product $j = 1 \ldots M$.

A positive influence $K_{ij} > 0$ is called excitation, a negative influence $K_{ij} < 0$ is referred to as inhibition. The latter is interpreted as **product cannibalisation**.

Implement That Model

"Under the Hood"

To implement this model we needed to overcome a few challenges:

- Ensuring a non-negative intensity $\rightarrow$ link function
- Integrating the intensity $\rightarrow$ numerical approximation
- Checking for stability $\rightarrow$ new criterion

All methodological details can be found in the mathematical draft on ArXiv [Deutsch and Ross, 2022].

Fit That Model on Real Data

We use a multivariate Hawkes Process where each dimension represents one product. This allows us to estimate the ‘influence’ K_{ij} from an event (sale) of one product i onto each product $j = 1 \ldots M$.

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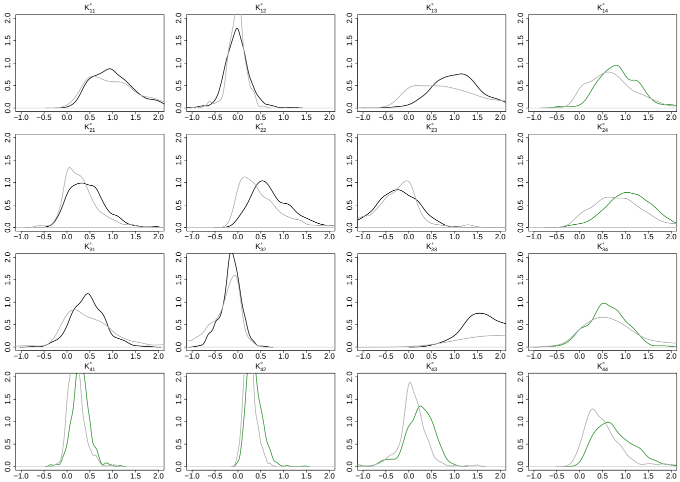

Figure 2: Posterior density estimates of $\mathbf{K}^*$ (excitation/inhibition parameter), using independent Normal (black) and horseshoe (grey) priors.

Summary

Methodological Advances
to make the implementation of a multivariate Hawkes Process with inhibition easier.

Formalisation of Product Cannibalisation
as a mathematical concept that can be estimated, monitored, and predicted.

[Deutsch and Ross, 2022]
arxiv.org/abs/2201.05009

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References I

- Copulsky, W. (1976).
  Cannibalism in the Marketplace.
  Journal of Marketing, 40(4):103–105.
- Daley, D. J. and Vere-Jones, D. (2003).
  An Introduction to the Theory of Point Processes: Volume I: Elementary Theory and Methods.
  Probability and Its Applications, An Introduction to the Theory of Point Processes. Springer-Verlag, New York, NY, 2 edition.
- Deutsch, I. and Ross, G. J. (2022).
  Bayesian Estimation of Multivariate Hawkes Processes with Inhibition and Sparsity.
  arXiv:2201.05009 [stat].
- Hawkes, A. G. (1971).
  Spectra of some self-exciting and mutually exciting point processes.
  Biometrika, 58(1):83–90.

References II

- Rizoiu, M.-A., Lee, Y., Mishra, S., and Xie, L. (2017).
  A Tutorial on Hawkes Processes for Events in Social Media.
  arXiv:1708.06401.